Automatica 164 (2024) 111645

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Delay-adaptive compensation for 3-D formation control of leader-actuated multi-agent systems[☆]

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A R T I C L E I N F O

Article history:
Received 9 February 2023
Received in revised form 24 November 2023
Accepted 25 February 2024
Available online 27 March 2024

Keywords:
Multi-agent system
Unknown input delay
PDE backstepping
Adaptive control
Formation control

A B S T R A C T

This paper focuses on the control of collective dynamics in large-scale multi-agent systems (MAS) operating in a 3-D space, with a specific emphasis on compensating for the influence of an unknown delay affecting the actuated leaders. The communication graph of the agents is defined on a mesh-grid 2-D cylindrical surface. We model the agents' collective dynamics by a complex- and a real-valued reaction–advection–diffusion 2-D partial differential equations (PDEs) whose states represent the 3-D position coordinates of the agents. The leader agents on the boundary suffer unknown actuator delay due to the cumulative computation and information transmission time. We design a delay-adaptive controller for the 2-D PDE by using PDE backstepping combined with a Lyapunov functional method, where the latter is employed to design an update law that generates real-time estimates of the unknown delay. Capitalizing on our recent result on the control of 1-D parabolic PDEs with unknown input delay, we use Fourier series expansion to bridge the control of 1-D PDEs to that of 2-D PDEs. To design the update law for the 2-D system, a new target system is defined to establish the closed-loop local boundedness of the system trajectories in H^2 norm and the regulation of the states to zero assuming a measurement of the spatially distributed plant's state. We illustrate the performance of the delay-adaptive controller by numerical simulations.

1. Introduction

Cooperative formation control in multi-agent systems (MAS) has garnered substantial interest due to its wide-ranging applications in various engineering domains, such as UAV formation flying (Alonso-Mora, Naegeli, Beardsley, & Beardsley, 2015), multi-robot collaboration (Alonso-Mora et al., 2019; Wang, Guo et al., 2017), vehicle queues (Fax & Murray, 2004), and satellite clusters (Zetocha et al., 2000). In MASs, communication delay, stemming from information exchange between agents, and input delays, arising from the processing/acquisition of data to update feedback control signals, can frequently lead to "suboptimal" performance and, in more critical cases, potentially result in system instability. Over the past few decades, a significant body of research in multi-agent systems focusing on communication delays has been produced. This research has predominantly employed high-order models and consensus protocols (Hou, Fu, Zhang, & Wu, 2017; Yu, Chen, & Cao, 2010). In the context of non-uniform communication delays, Lee and Spong (2006) establish the critical role of a globally reachable node in the information graph when designing linear agreement protocols for agents. The authors of Tian and Liu (2008) employ frequency domain analysis to derive a delay-dependent consensus condition for a first-order multi-agent system with input and communication delays. In Zhu and Jiang (2015), an event-triggered control is designed to establish a necessary and sufficient condition for leader-following consensus in multi-agent systems with input delays. A comparable control scheme for second-order consensus in multi-agent dynamical systems with input delays is introduced in Yu et al. (2010). Using the Artstein–Kwon–Pearson reduction method to convert delay-dependent systems into delay-free systems, fixed-time event-triggered consensus for linear MAS with input delay is achieved in Ai and Wang (2021). Based on a Lyapunov method for a mean square consensus problem of leader-following stochastic MAS with input time-dependent or constant delay, Tan, Cao, Li, and Alsaedi (2017) provides sufficient conditions to achieving

☆ The work was supported by the National Natural Science Foundation of China (62173084, 62303324, 61773112). Mamadou Diagne appreciates the support of the National Science Foundation of the USA under the grant number (FAIN): 2222250. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Rafael Vazquez under the direction of Editor Miroslav Krstic.

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https://doi.org/10.1016/j.automatica.2024.111645
0005-1098/

consensus. A solution for leader–follower consensus in nonlinear multi-agent systems with unknown non-uniform time-varying input delay is provided in Li, Hua, You, and Guan (2022) by constructing a delay-independent output-feedback controller for each follower. While the prevalent focus in the literature has been on the impact of input delay on follower agents, Qi, Wang, Fang, and Diagne (2019) addresses a known delay affecting the actuated leaders within a 3-D infinite-dimensional framework. Furthermore, most of these studies rely on ordinary differential equations (ODEs) models, namely, each agent's dynamic state is represented by an ODE, resulting in increased system complexity as the number of agents grows (Lee & Spong, 2006; Lin & Ren, 2014).

For multi-agent systems, control designs using partial differential equations (PDEs) provide a compact representation for capturing the dynamics of large-scale systems. These PDEs, whether they take a parabolic or hyperbolic form, describe the position coordinates of individual agents, as demonstrated in various works including Freudenthaler and Meurer (2020), Frihauf and Krstic (2011), Meurer and Krstic (2011), Qi, Vazquez, and Krstic (2015) and Qi, Zhang, and Ding (2018) and the reference therein. In the case of parabolic systems, the diffusion term, namely, the Laplace operator plays the role of MAS consensus protocol modeled by ODEs. Actuation of the leader agents positioned on the periphery of the communication structure demands a greater amount of information and computational resources compared to the follower agents. Consequently, leaders are more susceptible to delays that affect the formation control. Using the nominal delay-compensated boundary control law proposed in Krstic (2009) and Wang, Qi and Fang (2017), the authors of Qi et al. (2019) designed a boundary feedback law for MAS in 3-D space under a constant and known input delay. However, in practical scenarios, knowing precisely the value of the delay is often unfeasible, and instead, it is possible to estimate only its upper and lower bounds. To overcome such a challenge, the authors of Liu, Nojavanzadeh, Saberi, Saberi, and Stoorvogel (2021) investigate the determination of the delay bounds within which regulated state synchronization is attainable for a multi-agent system with unknown and nonuniform input delays. Similarly, in Zhang, Saberi, and Stoorvogel (2021), such a delay bound is characterized for semi-global state synchronization in a multi-agent system with actuator saturation and unknown nonuniform input delays. Nevertheless, there is a dearth of literature that addresses the issue of unknown delays in the context of multi-agent systems modeled by partial differential equations (PDEs). For reaction–diffusion systems subject to unknown boundary input delays (Wang, Qi, & Diagne, 2021) pioneering exploration led to a delay-adaptive compensated controller that ensures the regulation of the system's state to zero. Motivated by decontamination of a polluted surface, Wang, Diagne, and Qi (2022) constructed a delay-adaptive predictor feedback for reaction–diffusion systems subject to a delayed distributed input. The stabilization of deep-sea construction vessels using Batch-Least Squares Identifiers (Karafyllis, Kontorinaki, & Krstic, 2019) has been achieved in Wang and Diagne (2023) where finite-time exact identification of an unknown boundary input delay and simultaneously exponential regulation of the plant's state for a hyperbolic PDE–ODE system is ensured. More recently, a Lyapunov design approach that enables global stability for a hyperbolic PIDE (Partial Integro-Differential Equation) with an unknown boundary input delay was introduced in Wang, Qi and Krstic (2023).

We consider a formation control in 3-D space of a multi-agent system with unknown actuator delay. The collective dynamics of the Multi-Agent System (MAS) are characterized by two diffusion 2-D Partial Differential Equations (PDEs). The first PDE is complex-valued, with its states representing the agents' positions in the coordinates (x, y). The second PDE is real-valued, and its states correspond to the agents' positions in the coordinate z. We utilize a PDE backstepping design in tandem with a Lyapunov method to formulate a dynamic, delay-adaptive boundary feedback law. The nominal backstepping controller acquires complementary information about the unknown parameter through an update law driven by a carefully designed ODE. We introduce a Fourier series expansion to diminish the dimension of the 2-D system, transforming it into a set of n 1-D systems. In contrast to the result in Qi et al. (2019), the target system in the present study accounts for several highly nonlinear terms, generated by the delay-adaptive scheme, which pose challenges in establishing the convergence of their series representations, a crucial prerequisite for transforming the 1-D system into a 2-D system.

This paper is organized as follows. Section 2 introduces the PDE-based model for a MAS with actuation delay. Section 3 presents the delay-adaptive control design for the MAS collective dynamics subject to unknown actuation delay. The main result including the delay's adaptation law and the stability theorem is presented in Section 4. Section 5 gives the proof of the main result. Simulation results are provided in Section 6. The paper concludes with a discussion possible of future works in Section 7.

Notation: Throughout the paper, we adopt the following notation:

$$\Omega =\{(s,\theta):\ 0<s<1,\ \ -\pi\le\theta\le\pi\}, \tag{1}$$

$$\mathcal{D}_1 =\{(s,\tau):0\le\tau\le s\le 1\}, \tag{2}$$

$$\mathcal{D}_2 =\{(s,\tau):0\le s\le 1,\ 0\le\tau\le 1\}. \tag{3}$$

For $\chi:\Omega\to\mathbb{R}$, define the L^2, H^1 and H^2 norm as follows Tang, Qi, and Zhang (2017) and Qi et al. (2019)

$$\|\chi(s,\theta)\|^2_{L^2} :=\|\chi\|^2 := \int_0^1\int_{-\pi}^{\pi}|\chi(s,\theta)|^2\mathrm{d}\theta\mathrm{d}s,$$
$$\|\chi(s,\theta)\|^2_{H^1} :=\|\chi\|^2+\|\partial_s\chi\|^2+\|\partial_\theta\chi\|^2,$$
$$\|\chi(s,\theta)\|^2_{H^2} :=\|\chi\|^2_{H^1}+\|\partial_s^2\chi\|^2+2\|\partial_{s\theta}\chi\|^2+\|\partial_\theta^2\chi\|^2.$$

2. Multi-agent's PDEs model

2.1. Model description

Following Qi et al. (2019), we consider a group of agents located on a cylindrical surface undirected topology graph with index (i,j), $i=1,\dots,M$, $j=1,\dots,N$, moving in a 3-D space under the coordinate axes (x,y,z). A complex-valued state $u=x+\mathrm{j}y$ is defined to simplify the expression of the components on the (x,y) axes. Defining the discrete indexes (i,j) of the agents into Ω defined in (1), as $M,\ N\to\infty$ (see, Fig. 1), the continuum model of the collective dynamics of a large scale multi-agent system as follows

$$\partial_t u(s,\theta,t) =\Delta u(s,\theta,t)+\beta_1\partial_s u(s,\theta,t)+\lambda_1 u(s,\theta,t), \tag{4}$$

$$\partial_t z(s,\theta,t) =\Delta z(s,\theta,t)+\beta_2\partial_s z(s,\theta,t)+\lambda_2 z(s,\theta,t), \tag{5}$$

$$u(s,-\pi,t) =u(s,\pi,t),\quad u(0,\theta,t)=f_1(\theta), \tag{6}$$

$$u(1,\theta,t) =g_1(\theta)+U(\theta,t-D), \tag{7}$$

$$z(s,-\pi,t) =z(s,\pi,t),\quad z(0,\theta,t)=f_2(\theta), \tag{8}$$

$$z(1,\theta,t) =g_2(\theta)+Z(\theta,t-D), \tag{9}$$

where $(s,\theta,t)\in\Omega\times\mathbb{R}^+$, u, λ_1, $\beta_1\in\mathbb{C}$, z, λ_2, $\beta_2\in\mathbb{R}$. The coordinates (s,θ) are the spatial variables denoting the indexes of the agents in the continuum and Δ represents the following Laplace operator

$$\Delta u(s,\theta,t) = \partial_s^2 u(s,\theta,t)+\partial_\theta^2 u(s,\theta,t), \tag{10}$$

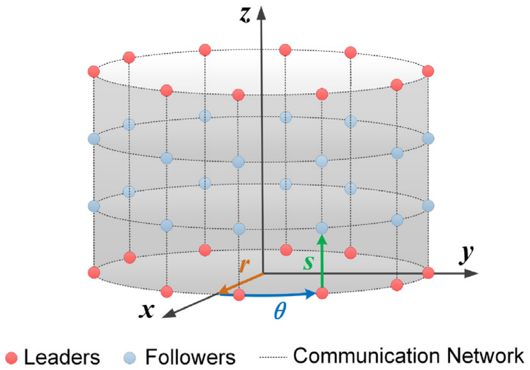

Fig. 1. Cylindrical surface topology prescribing the communication relationship among agents. The agents at the uppermost and lowermost layers are leaders. Each follower has four neighbors.

$$\Delta z(s,\theta,t)=\partial_s^2 z(s,\theta,t)+\partial_\theta^2 z(s,\theta,t), \tag{11}$$

which is defined as "consensus operators" for PDE representations (Ferrari-Trecate, Buffa, & Gati, 2006). Note that the boundary conditions (6) and (8) are periodical on the cylinder surface (see Fig. 1) while $f_1(\theta)$, $g_1(\theta)$, $f_2(\theta)$ and $g_2(\theta)$ are non-zero bounded boundary conditions for the states u and z, respectively.

To control the MAS to desired formations, we consider a configuration where the agents at the boundaries $s=0$ and $s=1$ are the leaders that drive all the agents to prescribe equilibrium. In (7) and (9), we defined the input delay $D>0$ affecting the actuated leaders and caused by communication lags in leader–follower configurations. In practice, the exact value of the delay is hard to measure, only the bounds of the unknown delay can be estimated, so we assume:

Assumption 1. Assume the bounds of the delay is known, i.e., $D\in\{D\in\mathbb{R}^+|\underline{D}\le D\le\overline{D}\}$, where $\underline{D}$ and $\overline{D}$ are the known lower and upper bounds, respectively.

Remark 1. Letting $\partial_t u(s,\theta,t)=0$ and $\partial_t z(s,\theta,t)=0$, one can solve (4)–(9) in the absence of control, resulting in the steady state profiles $\bar{u}(s,\theta)$ and $\bar{z}(s,\theta)$. These profiles correspond to the desired formations depending on the values of the parameters λ_1, β_1, λ_2, β_2, and the open-loop boundary conditions $f_1(\theta)$, $g_1(\theta)$, $f_2(\theta)$ and $g_2(\theta)$, as discussed in Qi et al. (2019).

3. Delay- adaptive boundary controller

First, define the error between the actual system and the desired system as $\tilde{u}(s,\theta,t)=u(s,\theta,t)-\bar{u}(s,\theta)$, and then introduce a change of variable $\phi(s,\theta,t)=e^{\frac{1}{2}\beta_1 s}\tilde{u}(s,\theta,t)$ for removing the convection term,

$$\partial_t\phi(s,\theta,t)=\Delta\phi(s,\theta,t)+\lambda_1'\phi(s,\theta,t),\quad s\in(0,1), \tag{12}$$

$$\phi(s,-\pi,t)=\phi(s,\pi,t),\quad \phi(0,\theta,t)=0, \tag{13}$$

$$\phi(1,\theta,t)=\Phi(\theta,t-D), \tag{14}$$

where $\lambda_1'=\lambda_1-\frac{1}{4}\beta_1^2$ and $\Phi(\theta,t-D)=e^{\frac{1}{2}\beta_1}U(\theta,t-D)$. By employing a transport PDE of ϑ representation of the delay appearing in (7), we transform the error system (12)–(14) as follows:

$$\partial_t\phi(s,\theta,t)=\Delta\phi(s,\theta,t)+\lambda_1'\phi(s,\theta,t),\quad s\in(0,1), \tag{15}$$

$$\phi(s,-\pi,t)=\phi(s,\pi,t),\quad \phi(0,\theta,t)=0, \tag{16}$$

$$\phi(1,\theta,t)=\vartheta(0,\theta,t), \tag{17}$$

$$D\partial_t\vartheta(s,\theta,t)=\partial_s\vartheta(s,\theta,t), \tag{18}$$

$$\vartheta(s,-\pi,t)=\vartheta(s,\pi,t),\quad \vartheta(1,\theta,t)=\Phi(\theta,t), \tag{19}$$

where $\vartheta(s,\theta,t)=\Phi(\theta,t+D(s-1))$, defined in $\Omega\times\mathbb{R}^+$. In the following, we will derive the dynamic boundary adaptive controller of the states u and z by employing the Fourier series expansion but limit our analysis to the u component of the state as a similar approach applies to the z dynamics.

3.1. Fourier series expansions

In order to transform the 2-D system (15)–(19) into n 1-D systems, we introduce the Fourier series expansion (Vazquez & Krstic, 2016) as

$$\phi(s,\theta,t)=\sum_{n=-\infty}^{\infty}\phi_n(s,t)e^{\mathrm{j}n\theta}, \tag{20}$$

$$\Phi(\theta,t)=\sum_{n=-\infty}^{\infty}\Phi_n(t)e^{\mathrm{j}n\theta}, \tag{21}$$

$$\vartheta(s,\theta,t)=\sum_{n=-\infty}^{\infty}\vartheta_n(s,t)e^{\mathrm{j}n\theta}, \tag{22}$$

where ϕ_n, Φ_n, ϑ_n are the n Fourier coefficients; independent of the angular argument θ. As an illustration, one of the coefficients in (20)–(22) is given as $\phi_n(s,t)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\phi(s,\psi,t)e^{-\mathrm{j}n\psi}\mathrm{d}\psi$. Substituting (20)–(22) into (15)–(19), we get the following 1-D PDE of the Fourier coefficients $\phi_n(s,t)$ and $\vartheta_n(s,t)$

$$\partial_t\phi_n(s,t)=\partial_s^2\phi_n(s,t)+(\lambda_1'-n^2)\phi_n(s,t),\ s\in(0,1), \tag{23}$$

$$\phi_n(0,t)=0,\quad \phi_n(1,t)=\vartheta_n(0,t), \tag{24}$$

$$D\partial_t\vartheta_n(s,t)=\partial_s\vartheta_n(s,t),\quad \vartheta_n(1,t)=\Phi_n(t). \tag{25}$$

In order to design feedback adaptive controller Φ_n, we postulate the following transformations

$$w_n(s,t)=\mathcal{T}_n[\phi_n](s,t)=\phi_n(s,t)-\int_0^s k_n(s,\tau)\phi_n(\tau,t)\mathrm{d}\tau, \tag{26}$$

$$h_n(s,t)=\mathcal{T}_n[\vartheta_n](s,t)=-\hat{D}(t)\int_0^s p_n(s,\tau,\hat{D}(t))\vartheta_n(\tau,t)\mathrm{d}\tau + \vartheta_n(s,t)-\int_0^1\gamma_n(s,\tau,\hat{D}(t))\phi_n(\tau,t)\mathrm{d}\tau, \tag{27}$$

whose the inverse transformations are

$$\phi_n(s,t)=\mathcal{T}_n^{-1}[w_n](s,t)=w_n(s,t)+\int_0^s l_n(s,\tau)w_n(\tau,t)\mathrm{d}\tau, \tag{28}$$

$$\vartheta_n(s,t)=\mathcal{T}_n^{-1}[h_n](s,t)=\hat{D}(t)\int_0^s q_n(s,\tau,\hat{D}(t))h_n(\tau,t)\mathrm{d}\tau + h_n(s,t)+\int_0^1\eta_n(s,\tau,\hat{D}(t))w_n(\tau,t)\mathrm{d}\tau, \tag{29}$$

where $\hat{D}(t)$ is the estimate of unknown input delay.[1] The kernels k_n, p_n, l_n, q_n are defined in $\mathcal{D}_1$, and γ_n, η_n are defined in $\mathcal{D}_2$, where $\mathcal{D}_1$ and $\mathcal{D}_2$ are defined in (2) and (3), respectively.

Hence, by PDE Backstepping method, (23)–(25) map into the following target system parameterized

$$\partial_t w_n(s,t)=\partial_s^2 w_n(s,t)-n^2 w_n(s,t), \tag{30}$$

$$w_n(0,t)=0,\quad w_n(1,t)=h_n(0,t), \tag{31}$$

$$D\partial_t h_n(s,t)=\partial_s h_n(s,t)-\tilde{D}P_{1n}(s,t)-D\dot{\hat{D}}P_{2n}(s,t), \tag{32}$$

$$h_n(1,t)=0, \tag{33}$$

[1] For the sake of simplicity, $\hat{D}(t)$ is defined as $\hat{D}$ in the remaining part of our developments.

where

$$P_{1n}(s,t)=\int_0^1\Big(-\partial_\tau\gamma_n(s,1,\hat{D})l_n(1,\tau)+\frac{1}{\hat{D}}\partial_s\gamma_n(s,\tau,\hat{D})+\frac{1}{\hat{D}}\int_\tau^1\partial_s\gamma_n(s,\tau,\hat{D})l_n(\xi,t)\mathrm{d}\xi\Big)w_n(\tau,t)\mathrm{d}\tau-\partial_\tau\gamma_n(s,1,\hat{D})h_n(0,t), \tag{34}$$

$$P_{2n}(s,t)=\int_0^1\Big(\int_\tau^1\partial_{\hat{D}}\gamma_n(s,\tau,\hat{D})l_n(\xi,\tau)\mathrm{d}\xi+\partial_{\hat{D}}\gamma_n(s,\tau,\hat{D})+\int_0^s(p_n(s,\xi,\hat{D})+\hat{D}\partial_{\hat{D}}p_n(s,\xi,\hat{D}))\eta_n(\xi,\tau,\hat{D})\mathrm{d}\xi\Big)\cdot w_n(\tau,t)\mathrm{d}\tau+\int_0^s\Big(\hat{D}\partial_{\hat{D}}p_n(s,\tau,\hat{D})+p_n(s,\tau,\hat{D})+\hat{D}\int_\tau^s(p_n(s,\xi,\hat{D})+\hat{D}\partial_{\hat{D}}p_n(s,\xi,\hat{D}))q_n(\xi,\tau,\hat{D})\mathrm{d}\xi\Big)\cdot h_n(\tau,t)\mathrm{d}\tau, \tag{35}$$

and $\tilde{D}=D-\hat{D}$. The mapping (26), (27) is well defined if the kernel functions $k_n(s,\tau)$, $\gamma_n(s,\tau)$ and $p_n(s,\tau)$ satisfy

$$\partial_s^2k_n(s,\tau)=\partial_\tau^2k_n(s,\tau)+\lambda_1'k_n(s,\tau), \tag{36}$$

$$k_n(s,0)=0,\quad k_n(s,s)=-\frac{\lambda_1'}{2}s, \tag{37}$$

$$\partial_s\gamma_n(s,\tau,\hat{D})=\hat{D}(\partial_\tau^2\gamma_n(s,\tau,\hat{D})+(\lambda_1'-n^2)\gamma_n(s,\tau,\hat{D})), \tag{38}$$

$$\gamma_n(s,0,\hat{D})=\gamma_n(s,1,\hat{D})=0,\quad \gamma_n(0,\tau,\hat{D})=k_n(1,\tau), \tag{39}$$

$$\partial_sp_n(s,\tau,\hat{D})=-\partial_\tau p_n(s,\tau,\hat{D}), \tag{40}$$

$$p_n(s,1,\hat{D})=-\partial_\tau\gamma_n(s,\tau,\hat{D})|_{\tau=1}. \tag{41}$$

The solution of the above gain kernels PDEs is given by

$$k_n(s,\tau)=-\lambda\tau\frac{I_1(\sqrt{\lambda_1'(s^2-\tau^2)})}{\sqrt{\lambda_1'(s^2-\tau^2)}}, \tag{42}$$

$$\gamma_n(s,\tau,\hat{D})=2\sum_{i=1}^\infty e^{\hat{D}(\lambda_1'-n^2-i^2\pi^2)s}\sin(i\pi\tau)\int_0^1\sin(i\pi\xi)\cdot k(1,\xi)\mathrm{d}\xi, \tag{43}$$

$$p_n(s,\tau,\hat{D})=-\partial_2\gamma_n(s-\tau,1,\hat{D}), \tag{44}$$

where $\partial_2\gamma_n(\cdot,\cdot,\cdot)$ denotes the derivative of $\gamma_n(\cdot,\cdot,\cdot)$ with respect to the second argument. Similarly, one can get the kernels in inverse transformations (28), (29):

$$l_n(s,\tau)=-\lambda\tau\frac{J_1(\sqrt{\lambda_1'(s^2-\tau^2)})}{\sqrt{\lambda_1'(s^2-\tau^2)}}, \tag{45}$$

$$\eta_n(s,\tau,\hat{D})=2\sum_{i=1}^\infty e^{-\hat{D}(n^2+i^2\pi^2)s}\sin(i\pi\tau)\int_0^1\sin(i\pi\xi)\cdot k(1,\xi)\mathrm{d}\xi, \tag{46}$$

$$q_n(s,\tau,\hat{D})=-\partial_2\eta_n(s-\tau,1,\hat{D}). \tag{47}$$

From (25), (27) and (33), the 1-D delay-compensated adaptive controller writes

$$\Phi_n(t)=\hat{D}\int_0^1p_n(1,\tau,\hat{D})\vartheta_n(\tau,t)\mathrm{d}\tau+\int_0^1\gamma_n(1,\tau,\hat{D})\phi_n(\tau,t)\mathrm{d}\tau. \tag{48}$$

3.2. 2-D delay-compensated adaptive controller

In order to obtain the 2-D delay-compensated adaptive controller, we assemble all the n 1-D transformations defined in (26)–(27) in the form of Fourier series to recover the 2-D domain components and then get

$$w(s,\theta,t)=\sum_{n=-\infty}^\infty w_n(s,t)e^{\mathrm{j}n\theta}=\phi(s,\theta,t)-\int_0^sk(s,\tau)\phi(\tau,\theta,t)\mathrm{d}\tau, \tag{49}$$

$$h(s,\theta,t)=\sum_{n=-\infty}^\infty h_n(s,t)e^{\mathrm{j}n\theta}=\vartheta(s,\theta,t)-\int_0^1\int_{-\pi}^\pi\gamma(s,\tau,\theta,\psi,\hat{D})w(\tau,\psi,t)\mathrm{d}\psi\mathrm{d}\tau-\hat{D}\int_0^s\int_{-\pi}^\pi p(s,\tau,\theta,\psi,\hat{D})\vartheta(\tau,\psi,t)\mathrm{d}\psi\mathrm{d}\tau, \tag{50}$$

where $k(s,\tau)$ is defined in (42), the related 2-D kernels are given as

$$\gamma(s,\tau,\theta,\psi,\hat{D})=2Q(s,\theta-\psi,\hat{D})\sum_{i=1}^\infty e^{\hat{D}(\lambda_1'-i^2\pi^2)s}\sin(i\pi\tau)\cdot\int_0^1\sin(i\pi\xi)k(1,\xi)\mathrm{d}\xi, \tag{51}$$

$$p(s,\tau,\theta,\psi,\hat{D})=-\partial_2\gamma(s-\tau,1,\theta-\psi,\hat{D}). \tag{52}$$

For all $s\in[0,1]$, defining $Q(s,\theta-\psi,\hat{D})=\frac{1}{2\pi}\sum_{n=-\infty}^\infty e^{-\hat{D}n^2s}\cdot e^{\mathrm{j}n(\theta-\psi)}$. Due to $0<Q(s,\theta-\psi,\hat{D})\le P(e^{-\hat{D}s},\theta-\psi)$, where P denotes Poisson Kernel. Using the properties of Poisson kernels (Brown & Churchill, 2009), one gets the boundedness of the kernel functions $\gamma(s,\tau,\theta,\psi,\hat{D})$ and $p(s,\tau,\theta,\psi,\hat{D})$. In a similar way, we get the inverse transformations of (49) and (50) are given by

$$\phi(s,\theta,t)=w(s,\theta,t)+\int_0^sl(s,\tau)w(\tau,\theta,t)\mathrm{d}\tau, \tag{53}$$

$$\vartheta(s,\theta,t)=\int_0^1\int_{-\pi}^\pi\eta(s,\tau,\theta,\psi,\hat{D})w(\tau,\psi,t)\mathrm{d}\psi\mathrm{d}\tau+h(s,\theta,t)+\hat{D}\int_0^s\int_{-\pi}^\pi q(s,\tau,\theta,\psi,\hat{D})h(\tau,\psi,t)\mathrm{d}\psi\mathrm{d}\tau, \tag{54}$$

where the gain kernels l, η and q are defined as

$$l(s,\tau)=-\lambda\tau\frac{J_1(\sqrt{\lambda_1'(s^2-\tau^2)})}{\sqrt{\lambda_1'(s^2-\tau^2)}}, \tag{55}$$

$$\eta(s,\tau,\theta,\psi,\hat{D})=2Q(s,\theta-\psi,\hat{D})\sum_{i=1}^\infty e^{-\hat{D}i^2\pi^2s}\sin(i\pi\tau)\cdot\int_0^1\sin(i\pi\xi)l(1,\xi)\mathrm{d}\xi, \tag{56}$$

$$q(s,\tau,\theta,\psi,\hat{D})=-\partial_2\eta(s-\tau,1,\theta,\psi,\hat{D}). \tag{57}$$

From (48), (51) and (52), we obtain the following delay-adaptive control law:

$$U(\theta,t)=\int_0^1\int_{-\pi}^\pi\gamma(1,\tau,\theta,\psi,\hat{D})e^{-\frac{1}{2}\beta_1(1-\tau)}(u(\tau,\psi,t)-\bar{u}(\tau,\psi))\mathrm{d}\psi\mathrm{d}\tau-\int_{t-D}^t\int_{-\pi}^\pi\partial_2\gamma(\frac{t-\nu}{\hat{D}},1,\theta,\psi,\hat{D})e^{\frac{1}{2}\beta_1}\cdot U(\psi,\nu)\mathrm{d}\psi\mathrm{d}\nu. \tag{58}$$

3.3. 2-D target system for the plant with unknown input delay

Similarly, in order to obtain the 2-D Target system for the plant with unknown input delay, we assemble all the n 1-D target systems defined in (30)–(33) in the form of Fourier series to return back to the 2-D domain

$$\partial_t w(s,\theta,t)=\Delta w(s,\theta,t), \tag{59}$$

$$w(s,-\pi,t)=w(s,\pi,t), \tag{60}$$

$$w(0,\theta,t)=0,\quad w(1,\theta,t)=h(0,\theta,t), \tag{61}$$

$$D\partial_t h(s,\theta,t)=\partial_s h(s,\theta,t)-\tilde{D}P_1(s,\theta,t)-D\dot{\hat{D}}P_2(s,\theta,t), \tag{62}$$

$$h(s,-\pi,t)=h(s,\pi,t),\quad h(1,\theta,t)=0, \tag{63}$$

with

$$P_1(s,\theta,t)=\int_0^1\int_{-\pi}^{\pi}M_1(s,\tau,\hat{\theta},\psi,t)w(\tau,\psi,t)\mathrm{d}\psi\mathrm{d}\tau + \int_{-\pi}^{\pi}M_2(s,\theta,\psi,t)h(0,\psi,t)\mathrm{d}\psi, \tag{64}$$

$$P_2(s,\theta,t)=\int_0^1\int_{-\pi}^{\pi}M_3(s,\tau,\theta,\psi,t)w(\tau,\psi,t)\mathrm{d}\psi\mathrm{d}\tau + \int_0^s\int_{-\pi}^{\pi}M_4(s,\tau,\theta,\psi,t)h(\tau,\psi,t)\mathrm{d}\psi\tau, \tag{65}$$

where M_i, $i=1,2,3,4$ are functions defined below:

$$M_1(s,\tau,\theta,\psi,t)=\frac{1}{\hat{D}}\left(\int_\tau^1\gamma_s(s,\xi,\theta,\psi,\hat{D})l(\xi,\tau)\mathrm{d}\xi + \gamma_s(s,\tau,\theta,\psi,\hat{D})\right)-\partial_2\gamma(s,1,\theta,\psi,\hat{D})l(1,\tau), \tag{66}$$

$$M_2(s,\tau,\theta,\psi,t)=-\partial_2\gamma(s,1,\theta,\psi,\hat{D}), \tag{67}$$

$$M_3(s,\tau,\theta,\psi,t)=\int_\tau^1\gamma_{\hat{D}}(s,\xi,\theta,\psi,\hat{D})l(\xi,\tau)\mathrm{d}\xi + \int_0^s\int_{-\pi}^{\pi}(p(s,\xi,\theta,\varphi,\hat{D})+\hat{D}p_{\hat{D}}(s,\tau,\theta,\varphi,\hat{D})) \cdot\eta(\xi,\tau,\varphi,\psi,\hat{D})\mathrm{d}\varphi\mathrm{d}\xi+\gamma_{\hat{D}}(s,\tau,\theta,\psi,\hat{D}), \tag{68}$$

$$M_4(s,\tau,\theta,\psi,t)=p(s,\tau,\theta,\psi,\hat{D})+\hat{D}p_{\hat{D}}(s,\tau,\theta,\psi,\hat{D}) + \hat{D}\int_\tau^s\int_{-\pi}^{\pi}(p(s,\xi,\theta,\varphi,\hat{D})+\hat{D}p_{\hat{D}}(s,\xi,\theta,\varphi,\hat{D})) \cdot q(\xi,\tau,\varphi,\psi,\hat{D})\mathrm{d}\varphi\mathrm{d}\xi. \tag{69}$$

4. The main result

To estimate the unknown parameter D, we construct the following update law

$$\dot{\hat{D}}=\varrho\mathrm{Proj}_{[\underline{D},\bar{D}]}\{\tau(t)\},\quad 0<\varrho<1, \tag{70}$$

where $\tau(t)$ is given as

$$\tau(t)=-2\int_0^1\int_{-\pi}^{\pi}(1+s)h(s,\theta,t)P_1(s,\theta,t)\mathrm{d}\theta\mathrm{d}s, \tag{71}$$

and the standard projection operator is defined as follows

$$\mathrm{Proj}_{[\underline{D},\bar{D}]}\{\tau(t)\}=\begin{cases}0 & \hat{D}=\underline{D}\ \textit{and}\ \tau(t)<0,\\ 0 & \hat{D}=\bar{D}\ \textit{and}\ \tau(t)>0,\\ \tau(t) & \textit{otherwise}.\end{cases} \tag{72}$$

Our claim is that the time-delayed multi-agent system studied in this paper achieves stable formation control, in other words, the state of the error system (15)–(19) tends to zero under the effect of the adaptive controller (58). The following theorem is established.

Theorem 1. *Consider the closed-loop system consisting of the plant (15)–(19), the control law (58), the updated law (70) under Assumption 1. Local boundedness and regulation of the system trajectories are guaranteed, i.e., there exist positive constants $\mathcal{M}_1$, $\mathcal{R}_1$ such that if the initial conditions $(\phi_0,\vartheta_0,\hat{D}_0)$ satisfy $\Psi_1(0)<\mathcal{M}_1$, where*

$$\Psi_1(t)=\|\phi\|_{H^2}^2+\|\partial_t\phi\|_{H^1}^2+\|\vartheta\|_{H^2}^2+\|\partial_{s\theta\theta}\vartheta\|^2+\|\partial_{ss\theta}\vartheta\|^2 +\|\vartheta(0,\cdot,t)\|^2+\|\partial_\theta\vartheta(0,\cdot,t)\|^2+\|\partial_\theta^2\vartheta(0,\cdot,t)\|^2 +\|\partial_t\vartheta(0,\cdot,t)\|^2+\|\partial_{t\theta}\vartheta(0,\cdot,t)\|^2+\tilde{D}^2, \tag{73}$$

the following holds:

$$\Psi_1(t)\le\mathcal{R}_1\Psi_1(0),\quad\forall t\ge 0; \tag{74}$$

furthermore,

$$\lim_{t\to\infty}\max_{(s,\theta)\in[0,1]\times[-\pi,\pi]}|\phi(s,\theta,t)|=0, \tag{75}$$

$$\lim_{t\to\infty}\max_{(s,\theta)\in[0,1]\times[-\pi,\pi]}|\vartheta(s,\theta,t)|=0. \tag{76}$$

Remark 2. Only local stability result is obtained due to the existence of the unbounded boundary input operator combined with the presence of highly nonlinear terms in the target system (59)–(63). In comparison to Wang et al. (2021), the need to ensure continuity of the communication topology of the multi-agent system in three-dimensional space leads to consider more complex norms of the system state (see. (73)) for the stability analysis.

5. Proof of the main result

We introduce the following change of variables

$$m(s,\theta,t)=w(s,\theta,t)-sh(0,\theta,t), \tag{77}$$

to create a homogeneous boundary condition of the target system

$$\partial_t m(s,\theta,t)=\Delta m(s,\theta,t)+s\partial_\theta^2 h(0,\theta,t)-s\partial_t h(0,\theta,t), \tag{78}$$

$$m(s,-\pi,t)=m(s,\pi,t),\quad m(0,\theta,t)=m(1,\theta,t)=0, \tag{79}$$

$$D\partial_t h(s,\theta,t)=\partial_s h(s,\theta,t)-\tilde{D}P_1(s,\theta,t)-D\dot{\hat{D}}P_2(s,\theta,t), \tag{80}$$

$$h(s,-\pi,t)=h(s,\pi,t),\quad h(1,\theta,t)=0, \tag{81}$$

with $w(s,\theta,t)$ in $P_i(s,\theta,t)$, $\{i=1,2\}$, is rewritten as $m(s,\theta,t)+sh(0,\theta,t)$.

We will prove Theorem 1 by

(1) proving the norm equivalence between the target system (78)–(81) and the error system (15)–(19) through Proposition 1,
(2) analyzing the local stability of the target system (78)–(81), and then deriving the stability of the error system based on norm equivalence's argument,
(3) and establishing the regulation of the state $\phi(s,\theta,t)$ and $\vartheta(s,\theta,t)$.

(1) Norm equivalence

We prove the equivalence between the error system (15)–(19) and target system (78)–(81) in the following Proposition.

Proposition 1. *The following estimates hold between the state of the error system (15)–(19), and the state of the target system (78)–(81):*

$$\begin{aligned}&\|\phi\|_{H^2}^2+\|\partial_t\phi\|_{H^1}^2+\|\vartheta\|_{H^2}^2+\|\partial_{s\theta\theta}\vartheta\|^2+\|\partial_{ss\theta}\vartheta\|^2\\&+\|\vartheta(0,\cdot,t)\|^2+\|\partial_\theta\vartheta(0,\cdot,t)\|^2+\|\partial_\theta^2\vartheta(0,\cdot,t)\|^2\\&+\|\partial_t\vartheta(0,\cdot,t)\|^2+\|\partial_{t\theta}\vartheta(0,\cdot,t)\|^2\\ \le&R_1(\|m\|_{H^2}^2+\|\partial_t m\|_{H^1}^2+\|h\|_{H^2}^2+\|\partial_{s\theta\theta}h\|^2+\|\partial_{ss\theta}h\|^2\\&+\|h(0,\cdot,t)\|^2+\|\partial_\theta h(0,\cdot,t)\|^2+\|\partial_\theta^2 h(0,\cdot,t)\|^2\end{aligned}$$

$$+\|\partial_t h(0,\cdot,t)\|^2+\|\partial_{t\theta} h(0,\cdot,t)\|^2), \quad (82)$$

$$(\|m\|_{H^2}^2+\|\partial_t m\|_{H^1}^2+\|h\|_{H^2}^2+\|\partial_{s\theta\theta} h\|^2+\|\partial_{ss\theta} h\|^2 + \|h(0,\cdot,t)\|^2+\|\partial_\theta h(0,\cdot,t)\|^2+\|\partial_\theta^2 h(0,\cdot,t)\|^2 + \|\partial_t h(0,\cdot,t)\|^2+\|\partial_{t\theta} h(0,\cdot,t)\|^2) \leq R_2(\|\phi\|_{H^2}^2+\|\partial_t\phi\|_{H^1}^2+\|\vartheta\|_{H^2}^2+\|\partial_{s\theta\theta}\vartheta\|^2+\|\partial_{ss\theta}\vartheta\|^2 + \|\vartheta(0,\cdot,t)\|^2+\|\partial_\theta\vartheta(0,\cdot,t)\|^2+\|\partial_\theta^2\vartheta(0,\cdot,t)\|^2 + \|\partial_t\vartheta(0,\cdot,t)\|^2+\|\partial_{t\theta}\vartheta(0,\cdot,t)\|^2), \quad (83)$$

where R_i, $i=1,2$ are sufficiently large positive constants.

The proof of Proposition 1 is stated in Appendix A of the supplementary material (Wang, Diagne and Qi, 2023), we omit here since the limit of the space.

Next, we show the local stability for the closed-loop system consisting of the (ϕ,ϑ)-system under the control law (58), and with the updated law (70)–(71).

(2) Local stability analysis

Since the error system (15)–(19) is equivalent to the target system (78)–(81), we establish the local stability of the target system by introducing the following Lyapunov–Krasovskii-type function,

$$V_1(t)=b_1(\|m\|_{H^2}^2+\|\partial_t m\|_{H^1}^2)+D\int_0^1\int_{-\pi}^{\pi}(1+s)(|h|^2 + |\partial_s h|^2+|\partial_\theta h|^2+|\Delta h|^2+|\partial_{s\theta\theta}h|^2+|\partial_{ss\theta}h|^2)\mathrm{d}\theta\mathrm{d}s + b_2D(\|h(0,\cdot,t)\|^2+\|\partial_\theta h(0,\cdot,t)\|^2+\|\partial_\theta^2 h(0,\cdot,t)\|^2 + \|\partial_t h(0,\cdot,t)\|^2+\|\partial_{t\theta}h(0,\cdot,t)\|^2)+\frac{\tilde{D}^2}{2\varrho}. \quad (84)$$

Taking the time derivative of (84), based on (64), (65), (77)–(81), and using Cauchy Schwartz's inequality, Young's inequality, Poincare's inequality, and integration by parts, we obtain that

$$\begin{aligned}\dot V_1(t)\leq& -b_1\left(\frac{3}{8}-\frac{1}{\sigma_2}-\frac{1}{\sigma_3}\right)\|m\|^2-2b_1\|\partial_\theta m\|^2 - \frac{b_1}{2}\|\partial_s m\|^2-b_1\left(2-\frac{1}{\sigma_1}-\frac{1}{\sigma_4}-\frac{1}{\sigma_5}\right)\|\Delta m\|^2-b_1\left(\frac{3}{8} - \frac{1}{\sigma_6}-\frac{1}{\sigma_7}\right)\|\partial_t m\|^2-\frac{b_1}{2}\|\partial_{ts}m\|^2-2b_1\|\partial_{t\theta}m\|^2-b_1(2 \\ &-\sigma_1-\sigma_8-\sigma_9)\|\Delta\partial_t m\|^2-(\|h\|^2+\|\partial_s h\|^2+\|\partial_\theta h\|^2 + \|\Delta h\|^2+\|\partial_{s\theta\theta}h\|^2+\|\partial_{ss\theta}h\|^2)-(1-b_2\sigma_{10})\|h(0,\cdot,t)\|^2 \\ &-\left(1-\frac{b_2}{\sigma_{10}}-\frac{3b_2\sigma_{13}}{D}-\frac{b_1(\sigma_3+\sigma_5)}{D^2}\right)\|\partial_s h(0,\cdot,t)\|^2 - \left(1-\frac{b_2}{\sigma_{11}}\right)\|\partial_\theta h(0,\cdot,t)\|^2-\left(1-\frac{7b_2}{D^3\sigma_{13}}-\frac{7b_1}{3D^4}(\sigma_7 + \frac{1}{\sigma_9})\right)\|\partial_s^2 h(0,\cdot,t)\|^2-\left(1-\frac{b_1(\sigma_2+\sigma_4)}{3}-\frac{b_2}{\sigma_{12}}\right) \\ &\cdot\|\partial_\theta^2 h(0,\cdot,t)\|^2-\left(1-\frac{7b_2}{D^3\sigma_{14}}\right)\|\partial_{ss\theta}h(0,\cdot,t)\|^2-\left(2 - b_2\sigma_{11}-\frac{3b_2\sigma_{14}}{D}\right)\|\partial_{s\theta}h(0,\cdot,t)\|^2-\left(1-\sigma_{12}b_2 - \frac{b_1}{D^2}(\sigma_6+\frac{1}{\sigma_8})\right)\|\partial_{s\theta\theta}h(0,\cdot,t)\|^2-\tilde{D}E_1(t)-D\dot{\hat{D}}E_2(t) \\ &+\tilde{D}^2E_3(t)+\dot{\hat{D}}^2E_4(t)+\ddot{\hat{D}}^2E_5(t)-\dot{\hat{D}}\frac{\tilde{D}}{\varrho}, \end{aligned} \quad (85)$$

where $\sigma_i>0$, $i=1,2,\ldots,14$, and

$$E_1(t)=2\int_0^1\int_{-\pi}^{\pi}(1+s)(hP_1+\partial_s h\partial_s P_1+\partial_\theta h\partial_\theta P_1 + \Delta h\partial_s^2P_1+\Delta h\partial_\theta^2 P_1+\partial_{s\theta\theta}h\partial_{s\theta\theta}P_1+\partial_{ss\theta}h\partial_{ss\theta}P_1)\mathrm{d}\theta\mathrm{d}s + 2b_2\int_{-\pi}^{\pi}(h(0,\theta,t)P_1(0,\theta,t)+\partial_\theta h(0,\theta,t)\partial_\theta P_1(0,\theta,t) + \partial_\theta^2h(0,\theta,t)\partial_\theta^2P_1(0,\theta,t))\mathrm{d}\theta\mathrm{d}s, \quad (86)$$

$$E_2(t)=2\int_0^1\int_{-\pi}^{\pi}(1+s)(hP_2+\partial_s h\partial_s P_2+\partial_\theta h\partial_\theta P_2 + \Delta h\partial_s^2P_2+\Delta h\partial_\theta^2 P_2+\partial_{s\theta\theta}h\partial_{s\theta\theta}P_2+\partial_{ss\theta}h\partial_{ss\theta}P_2)\mathrm{d}\theta\mathrm{d}s + 2b_2\int_{-\pi}^{\pi}(h(0,\theta,t)P_2(0,\theta,t)+\partial_\theta h(0,\theta,t)\partial_\theta P_2(0,\theta,t) + \partial_\theta^2h(0,\theta,t)\partial_\theta^2P_2(0,\theta,t))\mathrm{d}\theta\mathrm{d}s, \quad (87)$$

$$\begin{aligned}E_3(t)=&\left(\frac{b_1(\sigma_3+\sigma_5)}{D^2}+\frac{3b_2\sigma_{13}}{D}\right)\|P_1(0,\cdot,t)\|^2+\frac{b_1}{D^2}\left(\sigma_6 + \frac{1}{\sigma_8}\right)\|\partial_\theta^2P_1(0,\cdot,t)\|^2+\frac{3b_2\sigma_{14}}{D}\|\partial_\theta P_1(0,\cdot,t)\|^2 \\ &+7\left(\frac{b_1}{3D^2}(\sigma_7+\frac{1}{\sigma_9})+\frac{b_2}{\sigma_{14}D}\right)\left(\frac{1}{D^2}\|\partial_s P_1(0,\cdot,t)\|^2 + \|\partial_t P_1(0,\cdot,t)\|^2\right)+\frac{7b_2}{\sigma_{15}D}\left(\frac{1}{D^2}\|\partial_{s\theta}P_1(0,\cdot,t)\|^2 + \|\partial_{t\theta}P_1(0,\cdot,t)\|^2\right)+4(\|P_1(1,\cdot,t)\|^2+2\|\partial_\theta P_1(1,\cdot,t)\|^2 \\ &+\|\partial_\theta^2P_1(1,\cdot,t)\|^2)+12(\|\partial_s P_1(1,\cdot,t)\|^2+\|\partial_{s\theta}P_1(1,\cdot,t)\|^2 + D^2\|\partial_t P_1(1,\cdot,t)\|^2+D^2\|\partial_{t\theta}P_1(1,\cdot,t)\|^2),\end{aligned} \quad (88)$$

$$\begin{aligned}E_4(t)=&\left(\frac{7b_1}{3D^2}(\sigma_7+\frac{1}{\sigma_9})+\frac{7b_2}{D\sigma_{13}}\right)(\|P_1(0,\cdot,t)\|^2 + \|\partial_s P_2(0,\cdot,t)\|^2+D^2\|\partial_t P_2(0,\cdot,t)\|^2)+b_1\left(\sigma_6+\frac{1}{\sigma_8}\right) \\ &\cdot\|\partial_\theta^2P_2(0,\cdot,t)\|^2+\frac{7b_2}{D\sigma_{14}}(\|\partial_\theta P_1(0,\cdot,t)\|^2+\|\partial_{s\theta}P_2(0,\cdot,t)\|^2 + D^2\|\partial_{t\theta}P_2(0,\cdot,t)\|^2)+3b_2D\sigma_{14}\|\partial_\theta P_2(0,\cdot,t)\|^2+(b_1(\sigma_3 \\ &+\sigma_5)+3b_2D\sigma_{13})\|P_2(0,\cdot,t)\|^2+4D^2(\|P_2(1,\cdot,t)\|^2 + 2\|\partial_\theta P_2(1,\cdot,t)\|^2+\|\partial_\theta^2P_2(1,\cdot,t)\|^2)+12D^2(\|P_1(1,\cdot,t)\|^2 + \|\partial_s P_2(1,\cdot,t)\|^2+\|\partial_\theta P_1(1,\cdot,t)\|^2+D^2\|\partial_t P_2(1,\cdot,t)\|^2 \\ &+\|\partial_{s\theta}P_2(1,\cdot,t)\|^2+D^2\|\partial_{t\theta}P_2(1,\cdot,t)\|^2),\end{aligned} \quad (89)$$

$$E_5(t)=\left(\frac{7b_1}{3}(\sigma_7+\frac{1}{\sigma_9})+\frac{7b_2D}{\sigma_{13}}\right)\|P_2(0,\cdot,t)\|^2+\frac{7Db_2}{\sigma_{14}} \cdot\|\partial_\theta P_2(0,\cdot,t)\|^2+12D^4(\|P_2(1,\cdot,t)\|^2+\|\partial_\theta P_2(1,\cdot,t)\|^2). \quad (90)$$

By setting $\sigma_1=1$, $\sigma_2=\sigma_3=8$, $\sigma_4=\sigma_5=3$, $\sigma_6=\sigma_7=8$, $\sigma_8=\sigma_9=\frac{1}{3}$, $\sigma_{10}=\sigma_{11}=\sigma_{12}=\sigma_{13}=\sigma_{14}=1$, $0<b_1<\min\{\frac{3}{11},\frac{\underline{D}^2}{11},\frac{3\underline{D}^4}{77}\}$, $0<b_2<\min\{\frac{2\underline{D}}{3+\bar{D}},\frac{3-11b_1}{3},\frac{\underline{D}^2-11b_1}{\underline{D}^2},\frac{\underline{D}^3}{7},\frac{\underline{D}^2-11b_1}{\bar{D}(3+2\bar{D})},\frac{3\underline{D}^4-77b_1}{3\bar{D}(\bar{D}^3+7)}\}$, we get the following estimate

$$\dot V_1(t)\leq -\kappa_1V_2(t)-\tilde{D}E_1(t)-D\dot{\hat{D}}E_2(t)+\tilde{D}^2E_3(t) + \dot{\hat{D}}^2E_4(t)+\ddot{\hat{D}}^2E_5(t)-\dot{\hat{D}}\frac{\tilde{D}}{\varrho}, \quad (91)$$

where $\kappa_1=\min\{\frac{b_1}{8},\ 1-\frac{11b_1}{\underline{D}^2}-b_2-\frac{3b_2}{\underline{D}}\}>0$ and

$$V_2(t)=\|m\|_{H^2}^2+\|\partial_t m\|_{H^1}^2+\|h\|_{H^2}^2+\|\partial_{s\theta\theta}h\|^2+\|\partial_{ss\theta}h\|^2 + \|h(0,\cdot,t)\|^2. \quad (92)$$

With the help of Agmon's, Cauchy–Schwarz, and Young's inequalities, one can perform quite long calculations to derive the following estimates:

$$E_1(t)\leq 11L_1V_2(t),\qquad E_2(t)\leq 11L_1V_2(t), \quad (93)$$

$$E_3(t) \le (\alpha_1 + \alpha_2\theta^2 L_1^2 V_2(t)^2 + \tilde{D}^2\alpha_2) L_1 V_2(t), \tag{94}$$

$$E_4(t) \le (\alpha_3 + \alpha_4\theta^2 L_1^2 V_4(t)^2 + \tilde{D}^2\alpha_4) L_1 V_2(t), \tag{95}$$

$$E_5(t) \le \alpha_4 L_1 V_2(t), \quad \dot{\hat{D}}(t) \le \theta L_1 V_2(t), \tag{96}$$

$$\ddot{\hat{D}}(t) \le \theta(1 + \theta L_1 V_2(t) + |\tilde{D}|) L_1 V_2(t), \tag{97}$$

where

$$\alpha_1 = \frac{11(13\overline{D}^2 + 7)b_1}{3\underline{D}^4} + \frac{2(10\overline{D}^2 + 7)b_2}{\underline{D}^3} + 24\overline{D}^2 + 40, \tag{98}$$

$$\alpha_2 = \frac{77b_1}{3\underline{D}^2} + \frac{14b_2}{\underline{D}} + 24\overline{D}^2, \tag{99}$$

$$\alpha_3 = \frac{11(13\overline{D}^2 + 14)b_1}{3\underline{D}^2} + \frac{4(5\overline{D}^2 + 7)b_2}{\underline{D}} + 64\overline{D}^2 + 24\overline{D}^4, \tag{100}$$

$$\alpha_4 = \frac{77b_1}{3} + 14\overline{D}b_2 + 24\overline{D}^4, \tag{101}$$

and L_1 is a sufficiently large positive constant, which estimation method is similar to the method in Appendix A of the supplementary material (Wang, Diagne et al., 2023). And then, combining with (93)–(97), one can get

$$\begin{aligned}\dot{V}_1(t) \le& -\kappa_1 V_2(t) + |\tilde{D}|(8 + \frac{1}{\varrho}) L_1 V_2(t) + 8\overline{D} L_1^2 V_2(t)^2 \\ &+ \tilde{D}^2 \alpha_1 L_1 V_2(t) + \tilde{D}^2(2\alpha_2 + 12\alpha_4) L_1^3 V_2(t)^3 + \tilde{D}^4 \alpha_2 L_1 \\ &\cdot V_2(t) + (\alpha_3 + 12\alpha_4) L_1^3 V_2(t)^3 + 28\alpha_4 L_1^5 V_2(t)^5.\end{aligned} \tag{102}$$

From (84), it is easy to get $\tilde{D}^2 \le 2\varrho V_1(t) - 2\varrho\zeta_1 V_2(t)$, $\zeta_1 = \min\{b_1,\ \underline{D},\ b_2\underline{D}\}$. Using Cauchy–Schwarz's and Young's inequalities, one can deduce that

$$\left|\tilde{D}\right| \le \frac{\varepsilon_1}{2} + \frac{\tilde{D}^2}{2\varepsilon_1} \le \frac{\varepsilon_1}{2} + \frac{\varrho}{\varepsilon_1} V_1(t) - \frac{\varrho\zeta_1}{\varepsilon_1} V_2(t). \tag{103}$$

Again, using (84) we have

$$\zeta_1 V_0(t) \le V(t). \tag{104}$$

Substituting (103), (104) into (102), we derive the following estimate

$$\begin{aligned}\dot{V}_1(t) \le& -\left(\frac{\kappa_1}{2} - 8\varrho^2\alpha_2 L_1 V_1(t)^2\right) V_2(t) - \left(\frac{\kappa_1}{2} - L_1(8\right. \\ &\left. + \frac{1}{\varrho})(\frac{\varepsilon_1}{2} + \frac{\varrho}{\varepsilon_1} V_1(t)) - 2\varrho\alpha_1 L_1 V_1(t)\right) V_2(t) - L_1 \\ &\cdot \left(\frac{\varrho\zeta_1}{\varepsilon_1}(8 + \frac{1}{\varrho}) - (\frac{(\alpha_3 + 12\alpha_4)L_1^2}{\zeta_1} + 8\alpha_2\varrho^2\zeta_1^2) V_1(t)\right. \\ &\left. - 8\overline{D}L_1\right) V_2(t)^2 - 2\varrho L_1\left(\alpha_1\zeta_1 - \frac{(\alpha_2 + 13\alpha_4)L_1^2}{\zeta_1} V_1(t)^2\right) \\ &\cdot V_2(t)^2 - L_1^3\left(2\varrho(\alpha_2 + 13\alpha_4)\zeta_1 - \frac{28\alpha_4 L_1^2}{\zeta_1} V_1(t)\right) V_2(t)^4.\end{aligned} \tag{105}$$

Let ε_1 defined as $\varepsilon_1 < \min\left\{\frac{\kappa_1\varrho}{L_1(8\varrho+1)}, \frac{(8\varrho+1)\zeta_1}{8\varrho\overline{D}L_1}\right\}$, to ensure $V_1(0) \le \mu_1$, where

$$\begin{aligned}\mu_1 \triangleq \min&\left\{\frac{\varepsilon_1(\kappa_1\varrho - (8\varrho + 1)L_1\varepsilon_1)}{2\varrho L_1(8\varrho + 1 + 2\alpha_1\varrho\varepsilon_1)}, \frac{\sqrt{\kappa_1}}{4\varrho\sqrt{\alpha_2 L_1}},\right. \\ &\frac{\sqrt{\alpha_1}\zeta_1}{\sqrt{(\alpha_2 + 13\alpha_4)L_1}}, \frac{\varrho(\alpha_2 + 13\alpha_4)\zeta_1}{14\alpha_4 L_1^2}, \\ &\left.\frac{\zeta_1((8\varrho + 1)\zeta_1 - 8\overline{D}L_1\varepsilon_1)}{\varepsilon_1(4\varrho^2\alpha_2\zeta_1^2 + (\alpha_3 + 12\alpha_4)L_1^2)}\right\}.\end{aligned} \tag{106}$$

Therefore,

$$\begin{aligned}\dot{V}_1(t) \le& -(\delta_1(t) + \delta_2(t))V_2(t) - (\delta_3(t) + \delta_4(t))V_2(t)^2 \\ &+ \delta_5(t) V_2(t)^4,\end{aligned} \tag{107}$$

where

$$\delta_1(t) = \frac{\kappa_1}{2} - L_1(8 + \frac{1}{\varrho})(\frac{\varepsilon_1}{2} + \frac{\varrho}{\varepsilon_1} V_1(t)) - 2\varrho\alpha_1 L_1 V_1(t), \tag{108}$$

$$\delta_2(t) = \frac{\kappa_1}{2} - 4\varrho^2\alpha_2 L_1 V_1(t)^2, \tag{109}$$

$$\begin{aligned}\delta_3(t) = L_1&\left(\frac{\varrho\zeta_1}{\varepsilon_1}(8 + \frac{1}{\varrho}) - 8\overline{D}L_1 - (\frac{(\alpha_3 + 12\alpha_4)L_1^2}{\zeta_1}\right. \\ &\left. + 4\varrho^2\alpha_2) V_1(t)\right),\end{aligned} \tag{110}$$

$$\delta_4(t) = 2\varrho L_1\left(\alpha_1\zeta_1 - \frac{(\alpha_2 + 13\alpha_4)L_1^2}{\zeta_1} V_1(t)^2\right), \tag{111}$$

$$\delta_5(t) = L_1^3\left(2\varrho(\alpha_2 + 13\alpha_4)\zeta_1 - \frac{28\alpha_4 L_1^2}{\zeta_1} V_1(t)\right), \tag{112}$$

are nonnegative functions if the initial condition satisfies (106). Thus, $V_1(t) \le V_1(0),\ \forall t \ge 0$.

Using (82), we can get

$$\Psi_1(t) \le \frac{\max\{R_1, 1\}}{\min\{b_1, 2\underline{D}, b_2\underline{D}, \frac{1}{2\varrho}\}} V_1(t) \le \mu_2 V_1(0), \tag{113}$$

where $\Psi_1(t)$ is defined as (73), and $\mu_2 = \frac{\max\{R_1, 1\}}{\min\{b_1,\ \underline{D},\ b_2\underline{D},\ \frac{1}{2\varrho}\}}$. Hence, combining (106) and (113), we have $\mathcal{M}_1 = \mu_1\mu_2$.

From (83) and (84), one gets

$$V_1(t) \le \max\left\{\max\{b_1, 2\overline{D}, b_2\overline{D}\}R_2, \frac{1}{2\varrho}\right\}\Psi_1(t). \tag{114}$$

Knowing that $V_1(0) \le \max\{\max\{b_1, 2\overline{D}, b_2\overline{D}\}R_2, \frac{1}{2\varrho}\}\Psi_1(0)$, we arrive at (74) with $\mathcal{R}_1 = \mu_2\max\{\max\{b_1, 2\overline{D}, b_2\overline{D}\}R_2, \frac{1}{2\varrho}\}$, which proves the local stability of the closed-loop system.

Next, we will prove the regulation of the cascaded system (ϕ, ϑ) to complete the proof of Theorem 1.

(3) Regulation of the cascaded system

From (84) and (105), we get the boundedness of all terms in (92), and then, based on (82), we also get the boundedness of all terms of $\Psi_1(t)$. We will prove (75) and (76) in Theorem 1 by applying Lemma D.2 (Smyshlyaev & Krstic, 2010) to ensure the following facts:

- all terms in (92) are square integrable in time,
- $\frac{d}{dt}(\|m\|^2)$, $\frac{d}{dt}(\|h\|^2)$ and $\frac{d}{dt}(\|\partial_s h\|^2)$ are bounded.

Knowing that

$$\int_0^t \|m(\tau)\|^2 d\tau \le \frac{1}{\inf_{0\le\tau\le t}\delta_1(\tau)}\int_0^t \delta_1(\tau)V_2(\tau)d\tau, \tag{115}$$

and using (108), the following inequality holds:

$$\begin{aligned}\inf_{0\le\tau\le t}\delta_1(t) =& \frac{\kappa_1}{2} - L_1(8 + \frac{1}{\varrho})\left(\frac{\varepsilon_1}{2} + \frac{\varrho}{\varepsilon_1}V_1(t)\right) \\ &- 2\varrho\alpha_1 L_1 V_1(t).\end{aligned} \tag{116}$$

Since $\dot{V}_1 \le -(\delta_1(t)+\delta_2(t))V_2(t)-(\delta_3(t)+\delta_4(t))V_1(t)^2+\delta_5(t)V_2(t)^4$ and $\delta_i(t)$ are nonnegative functions, we have $\dot{V}_1 \le -\delta_1(t)V_2(t)$, and integrating it over $[0,\ t]$ leads to

$$\int_0^t \delta_1(\tau)V_2(\tau)d\tau \le V_1(0) \le \mu_1. \tag{117}$$

Substituting (116) and (117) into (115), we get $\|m\|$ is square integrable in time. Similarly, one can establish that other terms in (92) are square-integrable in time.

To prove that $\frac{d}{dt}(\|m\|^2)$, $\frac{d}{dt}(\|h\|^2)$ and $\frac{d}{dt}(\|\partial_s h\|^2)$ are bounded, we define the Lyapunov function

$$V_3(t) = \frac{1}{2}\|m\|^2 + \frac{b_3 D}{2}\int_0^1\int_{-\pi}^{\pi}(1+s)(|h|^2+|\partial_s h|^2)\mathrm{d}\theta\mathrm{d}s, \quad (118)$$

where b_3 is a positive constant. Taking the derivative of (118) with respect to time, and using integration by parts and Young's inequality, the following holds

$$\begin{aligned}\dot V_3(t) \le & -\|\partial_s m\|^2 - b_3\|h\|^2 - b_3\|\partial_s h\|^2 + (\frac{1}{2\iota_7}+\frac{1}{2\iota_8})\|m\|^2 \\ & + \frac{\iota_7}{6}\|\partial_{\theta\theta} h\|^2 + \frac{\iota_7}{6}\|\partial_{\theta\theta s} h\|^2 + \frac{\iota_8}{2\underline{D}^2}|\tilde D|^2\|P_1(0,\cdot,t)\|^2 \\ & - (\frac{b_3}{2} - \frac{\iota_8}{2\underline{D}^2})\|\partial_s h(0,\cdot,t)\|^2 + \frac{\iota_8}{2}|\dot{\hat D}|^2\|P_2(0,\cdot,t)\|^2 + 4b_3 \\ & \cdot |\tilde D|^2\|h\|\|P_1\| + 2b_3|\tilde D|^2\|P_1(1,\cdot,t)\|^2 + 4b_3|\tilde D|\|h\|\|P_1\| \\ & + 2b_3\overline{D}^2|\dot{\hat D}|^2\|P_2(1,\cdot,t)\|^2 + 4b_3|\tilde D|^2\|\partial_s h\|\|\partial_s P_1\| \\ & + 4b_3|\tilde D|\|h\|\|P_2\| + 4b_3\overline{D}|\dot{\hat D}|\|\partial_s h\|\|\partial_s P_2\|. \quad (119)\end{aligned}$$

Setting $\iota_7 = \iota_8 = 8$ and $b_3 > \frac{4}{\underline{D}^2}$, we have

$$\dot V_3 \le -c_1 V_3 + f_1(t)V_3 + f_2(t) < \infty, \quad (120)$$

where we use Young's and Agmon's inequalities, $c_1 = \min\{\frac{1}{4}, \frac{1}{2\overline{D}}\}$, and

$$f_1(t) = \frac{2\tilde D^2}{\underline{D}}(\tilde D^2 + \overline{D}^2|\dot{\hat D}|^2), \quad (121)$$

$$\begin{aligned}f_2(t) = & \frac{4}{3}(\|\partial_{\theta\theta}h\|^2 + \|\partial_{\theta\theta s}h\|^2) + 2b_3|\tilde D|^2\|P_1(1,\cdot,t)\|^2 \\ & + 2b_3\overline{D}^2|\dot{\hat D}|^2\|P_2(1,\cdot,t)\|^2 + \frac{4|\tilde D|^2}{\underline{D}^2}\|P_1(0,\cdot,t)\|^2 + 4|\dot{\hat D}|^2 \\ & \cdot \|P_2(0,\cdot,t)\|^2 + 2b_3|\tilde D|^2\|P_1(1,\cdot,t)\|^2 + 2b_3\|P_1\|^2 \\ & + 2b_3\|\partial_s P_1\|^2 + 2b_3\overline{D}^2|\dot{\hat D}|^2\|P_2(1,\cdot,t)\|^2 + 2b_3\|P_2\|^2 \\ & + 2b_3\|\partial_s P_2\|^2. \quad (122)\end{aligned}$$

Combining (64) and (65), we get that $|\dot{\hat D}|$, $\|P_1(0,\cdot,t)\|^2$, $\|P_2(0,\cdot,t)\|^2$, $\|P_1(1,\cdot,t)\|^2$, $\|P_2(1,\cdot,t)\|^2$, $\|P_1\|^2$ and $\|P_2\|^2$ are bounded and integrable. Thereby, $f_1(t)$ and $f_2(t)$ are bounded and integrable functions of time. Thus, from (120), we deduce that $\dot V_2 \le \infty$, which proves the boundedness of $\frac{d}{dt}(\|m\|^2)$, $\frac{d}{dt}(\|h\|^2)$ and $\frac{d}{dt}(\|\partial_s h\|^2)$. Moreover, by Lemma D.2 (Smyshlyaev & Krstic, 2010), it holds that $\|m\|$, $\|h\|$, $\|\partial_s h\| \to 0$ as $t\to\infty$. Knowing that $\|h(0,\cdot,t)\|^2 \le 2\|h\|\|\partial_s h\|$, so $\|h(0,\cdot,t)\|^2 \to 0$ as $t\to\infty$. From (53) and (77), one can get

$$\|\phi\|^2 \le 4(1+\|l(s,\tau)\|^2)\|m\|^2 + 4\|l(s,\tau)\|^2(\|h\|^2+\|\partial_s h\|^2). \quad (123)$$

So, we get $\|\phi\|^2 \to 0$ as $t\to\infty$. Since $\|\phi\|_{H^2}$ is bounded, we can get $\phi(s,\theta,t)^2 \le C\|\phi\|\|\phi\|_{H^2}$ by using Agmon's inequality, and then we get $\phi(s,\theta,t)$ is regulated. Similarly, we can get $\vartheta(s,\theta,t)$ is also regulated.

6. Numerical simulations

6.1. Control laws for the leaders and the followers

In order to implement control laws of the followers, we discretize the PDEs (4) and (5). For $u\in\Omega$, we define the following discretized grid

$$s_i = (i-1)h_s, \quad \theta_j = (j-1)h_\theta, \quad d_k = (k-1)\Delta D, \quad (124)$$

for $i = 2,\ldots,M-1$, $j = 1,\ldots,N$, $k = 1,\ldots,M'$, where $h_s = \frac{1}{M-1}$, $h_\theta = \frac{2\pi}{N-1}$ and $\Delta D = \frac{D}{M'-1}$. Using a three-point central difference approximation, the control laws of the follower agents (i,j) are written as

$$\begin{aligned}\dot u_{ij} = & \frac{(u_{i+1,j}-u_{i,j})-(u_{i,j}-u_{i-1,j})}{h_s^2} + \beta_1\frac{u_{i+1,j}-u_{i-1,j}}{2h_s} \\ & + \frac{(u_{i,j+1}-u_{i,j})-(u_{i,j}-u_{i,j-1})}{h_\theta^2} + \lambda_1 u_{i,j}, \quad (125)\end{aligned}$$

where $i = 2,\ldots,M-1$, $j = 1,\ldots,N$, and all the state variables in θ space are 2π periodic, namely, $u_{i,1} = u_{i,N}$. The leader agents with guiding role at the boundary $s = 0$, namely $i = 1$ are formed as $u_{1,j} = f_1(\theta_j)$. For the leader agents at the boundary $s = 0$, namely $i = M$, from the discretized form of (58), the state feedback control action is given by

$$\begin{aligned}u_{M,j}(t) = & \sum_{m=1}^{M}\sum_{l=1}^{N} a_{m,l}\gamma_{j,m,l}e^{-\frac{1}{2}\beta_1(1-s_m)}(u_{m,l}(t)-\bar u_{m,l}(t)) \\ & - \sum_{k=1}^{M'}\sum_{l=1}^{N} a'_{k,l}\gamma'_{j,k,l}u_{M,l}(t-D+d_k) + \bar u_{M,j}, \quad (126)\end{aligned}$$

where $\gamma_{j,m,l}$ and $\gamma'_{j,k,l}$ can be discretized from (51) and (52). M, N, and M' are odd numbers according to Simpson's rule. The control laws for the z-coordinate can be obtained in a similar way.

6.2. Simulation results

A formation control simulation example with 51×50 agents on a mesh grid in the 3-D space illustrates the performance of the proposed control laws with unknown input delay. The real value of input delay $D = 2$, and the upper and lower bounds of the unknown delay are $\underline{D} = 0.1$ and $\overline{D} = 4$, respectively. The adaptive gain is fixed at $\varrho = 0.05$. The model's parameters are $\lambda_1 = \lambda_2 = 10$, $\beta_1 = \beta_2 = 0$. The control goal is to drive the formation of the agents from an initial equilibrium state characterized by the boundary values $f_1(\theta) = -e^{j\theta} + e^{-j2\theta}$, $g_1(\theta) = e^{j\theta} - e^{-j2\theta}$, $f_2(\theta) = -1.9$, $g_2(\theta) = 1.9$ and the parameters $\lambda_1 = \lambda_2 = 10$, $\beta_1 = \beta_2 = 0$ to a desired formation with boundary $f_1(\theta) = g_1(\theta) = e^{j\theta}$, $f_2(\theta) = 0$, $g_2(\theta) = 1.3$ and the parameters of $\lambda_1 = 30$, $\lambda_2 = 20$, $\beta_1 = \beta_2 = 1$. Fig. 2 shows the formation diagram (or snapshots of the evolution in time) of a 3-D multi-agent formation with an initial value of the unknown delay estimate $\hat D = 4$ and from the initial to the desired formation. The six snapshots of the formation's state illustrate the smooth evolution of collective dynamics between two different reference formations when the input delays are unknown. Fig. 3 shows the time-evolution of the control signals, and it is clear that the control effort tends to zero and ensures the stability of the closed-loop system dynamic. In Fig. 4, (a) shows the dynamics of the update rate of the unknown parameter, $\dot{\hat D}$, when its initial value is $\hat D(0) = 4$. It is clear that the updated rate gradually tends to zero over time; (b) describes the estimate of the unknown input delay for the system subject to the designed adaptive control law for a given initial value $\hat D(0) = 4$: the estimated delay $\hat D$ gradually converges to the real value $D = 2$. In Fig. 5, (a) and (b) show the tracking error of agents indexed by $i = 5$, $i = 15$, $i = 30$, and $i = 51$ (actuator leaders) and the average of all agents on the horizontal and vertical directions, respectively, under non-adaptive boundary control. It can be seen from the figure that the tracking error gradually tends to 0 with time evolution. Figures (c) and (d) show the L^2-norm of average tracking error of all the agents in the horizontal and vertical directions, respectively. It can be seen that if the estimate of unknown delay $\hat D$ does not match the true value of the delay $D = 2$, namely if a delay mismatch occurs, the tracking error diverges.

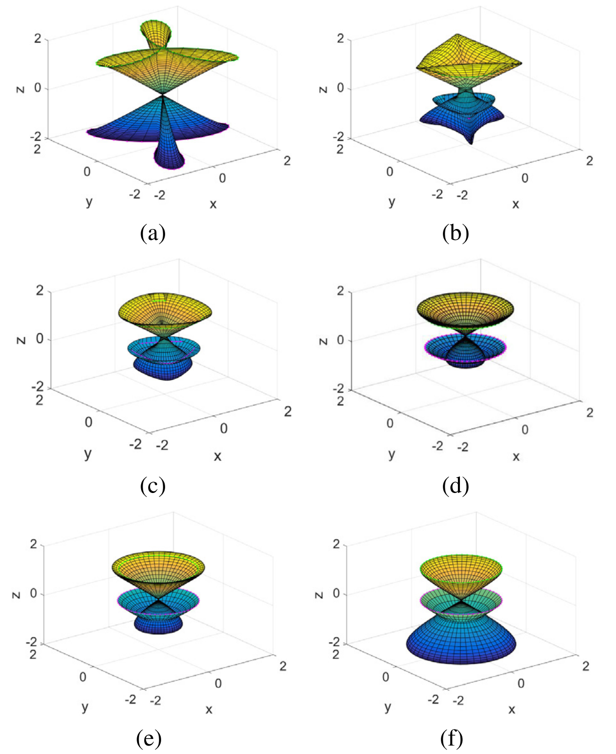

Fig. 2. The adaptive formation change process of the multi-agent system with unknown delay initial value $\hat{D}(0) = 4$. (a) $t = 0$ s (b) $t = 0.09$ s (c) $t = 0.2$ s (d) $t = 2$ s (e) $t = 4$ s (f) $t = 40$ s.

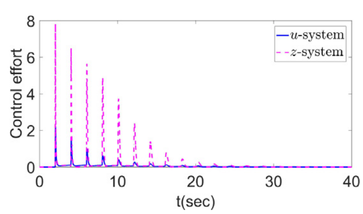

Fig. 3. Time-evolution of the control signals.

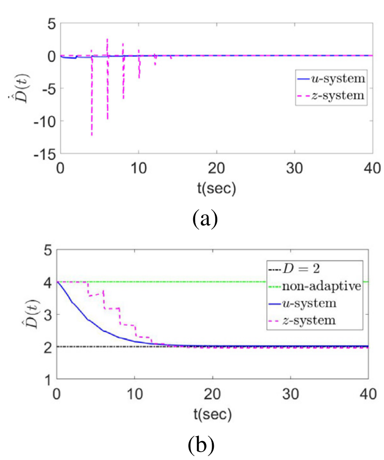

Fig. 4. Delay estimate. (a) Dynamics of the updated law $\dot{\hat{D}}(t)$ (b) Time-evolution of the estimate of the unknown parameter $\hat{D}(t)$.

7. Conclusion

This paper studies the formation control of MAS with unknown input delay in 3-D space via cylindrical topology. To achieve the desired 3-D formation with stable transitions, we propose an adaptive controller with the backstepping method. The update law for estimating the unknown parameter is designed using the Lyapunov method. As the dimensionality increases, the complexity of the problem grows significantly. To address this, we introduce a Fourier series to transform the PDE describing the two-dimensional cylindrical communication topology into the sum of infinite one-dimensional systems. Subsequently, we prove the local stability of the closed-loop system and the regulation of the system's state to zero by a rather intricate Lyapunov function. In future work, we will extend our research to the systems subject to both unknown plant coefficients and input delays.

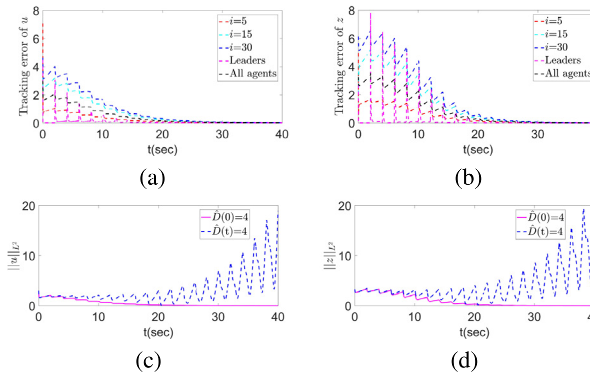

Fig. 5. (a) Tracking error of u system under adaptive control, (b) Observation error of z system under adaptive control, (c) Average tracking error of u system, (d) Average tracking error of z system.

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