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Estimation of boundary parameters in general heterodirectional linear hyperbolic systems[☆]

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ABSTRACT

In this paper, we extend recent results on state and boundary parameter estimation in coupled systems of linear partial differential equations (PDEs) of the hyperbolic type consisting of n rightward and one leftward convecting equations, to the general case which involves an arbitrary number of PDEs convecting in both directions. Two adaptive observers are derived based on swapping design, where one introduces a set of filters that can be used to express the system states as linear, static combinations of the filter states and the unknown parameters. Standard parameter identification laws can then be applied to estimate the unknown parameters. One observer which requires sensing at both boundaries, generates estimates of the boundary parameters and system states, while the second observer estimates the parameters from sensing limited to the boundary anti-collocated with the uncertain parameters. Proof of boundedness of the adaptive laws is offered, and sufficient conditions ensuring exponential convergence are derived. The theory is verified in simulations.

1. Introduction

1.1. Background

Relevant physical systems that can be modeled as hyperbolic partial differential equations (PDEs) are, to mention a few, heat exchangers (Xu & Sallet, 2010), transmission lines (Curró, Fusco, & Manganaro, 2011), road traffic (Amin, Hante, & Bayen, 2008), oil wells (Landet, Pavlov, & Aamo, 2013) and multiphase flow (Di-agne, Diagne, Tang, & Krstić, 2017; Di Meglio, 2011) and time-delays (Krstić & Smyshlyaev, 2008). These distributed parameter systems give rise to important estimation and control problems, for which early results can be found in Coron, Novel, and Bastin (2007), Greenberg and Tsien (1984) and Litrico and Fromion (2006), and more recently in Gou and Jin (2015), which deals with the boundary disturbance rejection problem for linear wave PDEs based on the so-called ADRC methodology.

Rigorous designs have recently started to appear based on the so-called backstepping approach. The key ingredient of the backstepping technique is the introduction of an invertible Volterra transformation that maps the original system of PDEs into a simpler target system whose stability is easier to establish. Then the invertibility of the backstepping transformation allows to state the stability properties for the original system from the target system analysis. The aforementioned framework represents a major shift for infinite dimensional controller design. Backstepping applied to infinite dimensional systems was initially developed for parabolic PDEs (Bosković, Krstić, & Liu, 2001), with the first version in its infinite-dimensional form presented in Liu (2003). The method has later been applied to e.g. fluid flows (Aamo, Smyshlyaev, Krstić, & Foss, 2004), nonlinear parabolic equations (Vazquez & Krstić, 2008a,b) and for the boundary stabilization of a one-dimensional wave equation with an internal spatially varying antidamping term in Smyshlyaev, Cerpa, and Krstić (2010). The extension to first order hyperbolic PDEs was done in Krstić and Smyshlyaev (2008), with an expansion to 2×2 systems in Vazquez, Krstić, and Coron (2011) and $n + 1$ systems in Di Meglio, Vazquez, and Krstić (2013). In such systems, n PDEs convect in the same direction, with a single one convecting in the opposite

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direction. The general $m+n$ case with an arbitrary number of PDEs convecting in each direction was given in Hu, Di Meglio, Vazquez, and Krstić (2016), and therefore presented the definitive solution to the problem of controlling coupled hyperbolic PDEs.

Parabolic PDEs were adaptively stabilized using output feedback in Smyshlyaev and Krstić (2007), while a comprehensive set of solutions for adaptive control of general linear parabolic PDEs given in Smyshlyaev and Krstić (2010), using both full-state and output-feedback, and employing three approaches for the design of parameter estimators and adaptive observer.

The efforts yielding results on adaptive control of general first-order non-local hyperbolic PDEs were launched in Bernard and Krstić (2014), where a hyperbolic partial integro-differential equation was adaptively stabilized using boundary sensing only. Also, Xu and Liu (2016) consider a subclass of systems considered in Bernard and Krstić (2014) and present a full-state feedback solution, with the implementation of the proposed controller requiring an online computation of the controller gains due to the time dependency of the adaptive kernel functions.

An adaptive observer for hyperbolic systems was also derived in Di Meglio, Bresch-Pietri, and Aarsnes (2014), where additive disturbance terms in the boundary conditions were estimated. The derived method was applied to a problem from underbalanced drilling in the oil industry, estimating uncertain parameters. Additionally, in Tang and Krstić (2014), backstepping was used in conjunction with sliding mode control to design an adaptive controller estimating and taking into account an uncertain parameter in the boundary condition at the same boundary as actuation. We mention that significant results dealing with observers design for disturbance rejection in distributed parameter systems were discussed in Aamo (2013), Anfinsen and Aamo (2015) and Anfinsen, Di Meglio, and Aamo (2016) and especially, the leak detection problem in pipe flows is successfully resolved in Aamo (2016). It is important to notice that these results can be substantially interpreted at a fundamental level as adaptive observers for hyperbolic PDEs.

Kreisselmeier filters (K-filters) are often used for both finite and infinite dimensional systems estimation related problems. The key feature of using K-filters is to derive a static relationship between the system states and the unknown parameters. The static relationship helps to build Lyapunov-based update laws that ensure the convergence of the unknown parameter estimates. We mention that this method was later named swapping design and was more thoroughly investigated in Krstić, Kanellakopoulos, and Kokotović (1995) for ODEs and extended to PDEs of parabolic type in Smyshlyaev and Krstić (2010).

1.2. Contribution

In the present work, we investigate the problem of estimating unknown additive and multiplicative parameters in the boundary condition of a general system of coupled 1-D first order hyperbolic PDEs, with an arbitrary number of PDEs convecting in each direction. Two main estimation problems are solved:

- First off, we estimate both the unknown parameters and the system states when allowing sensing at both boundaries.
- Secondly, we restrict the sensing to be anti-collocated with the parameter uncertainty, and generate estimates of the unknown parameters, as well as estimates of the system states from an earlier point in time which precedes the real time by a time interval that corresponds to the fastest of the transport speeds that separate the boundary where sensing takes place from the boundary at which the uncertainty is present.

The former of these two problems was independently solved for the $n+1$ case in Anfinsen, Diagne, Aamo, and Krstić (2016) and Bin and Di Meglio (in press). The non-adaptive version of the latter problem follows from the results in Hu et al. (2016) using a transformation, and under the assumption of having known multiplicative parameters, Aamo (2013) and Anfinsen and Aamo (2015) investigated the latter problem for 2×2 systems, considering an unknown (possibly time varying) additive disturbance term entering at the boundary. Moreover, the observers designed in Aamo (2013) and Anfinsen and Aamo (2015) allow for the estimation of the system states. The same problem is successfully tackled in Di Meglio et al. (2014) for the general $n+1$ system. One shall mention the recent contribution (Anfinsen, Di Meglio et al., 2016) which achieved the estimation of both multiplicative and additive disturbances for the $n+1$ case using a swapping design based observer with sensing restricted to be anti-collocated with the unknown parameters, and also producing estimates of the system states from an earlier point in time which precedes the real time by a time interval that corresponds to the fastest of the transport speeds that separate the boundary where sensing takes place from the boundary at which the uncertainty is present. Our contribution is the extension of the results in Anfinsen, Diagne et al. (2016) and Anfinsen, Di Meglio et al. (2016) to the general $m+n$ case.

1.3. Organization

This paper is organized as follows: in Section 2 we present the dynamical systems and state the two different estimation problems to be investigated. The first problem concerning estimation of unknown parameters in one boundary condition, as well as estimating the system states is solved in Section 3. In Section 4, we propose a solution to the second problem concerning estimation of boundary condition parameters from sensing restricted to be anti-collocated from the uncertain parameters. Demonstrations through simulations are offered in Section 5, with some concluding remarks given in Section 6.

2. Problem statements

We consider the system described by the following linear hyperbolic PDEs

$$u_t(x,t) + \Lambda^+ u_x(x,t) = \Sigma^{++}u(x,t) + \Sigma^{+-}v(x,t) \tag{1}$$

$$v_t(x,t) - \Lambda^- v_x(x,t) = \Sigma^{-+}u(x,t) + \Sigma^{--}v(x,t) \tag{2}$$

$$u(0,t) = Q_0 v(0,t) + d \tag{3}$$

$$v(1,t) = C_1 u(1,t) + U(t) \tag{4}$$

where

$$u(x,t) = \begin{bmatrix} u_1(x,t) & u_2(x,t) & \cdots & u_n(x,t)\end{bmatrix}^T \tag{5}$$

$$v(x,t) = \begin{bmatrix} v_1(x,t) & v_2(x,t) & \cdots & v_m(x,t)\end{bmatrix}^T \tag{6}$$

are the systems states, and

$$\Lambda^+ = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\} \tag{7}$$

$$\Lambda^- = \text{diag}\{\mu_1, \mu_2, \ldots, \mu_m\} \tag{8}$$

are the transport speeds, subject to the restriction

$$-\mu_1 < \cdots < -\mu_m < 0 < \lambda_1 < \lambda_2 < \cdots < \lambda_n. \tag{9}$$

The in-domain parameters are given as

$$\Sigma^{++} = \{\sigma_{ij}^{++}\}_{1\le i\le n, 1\le j\le n} \tag{10}$$

$$\Sigma^{+-} = \{\sigma_{ij}^{+-}\}_{1\le i\le n, 1\le j\le m} \tag{11}$$

$$\Sigma^{-+} = \{\sigma_{ij}^{-+}\}_{1\le i\le m, 1\le j\le n} \tag{12}$$

$$\Sigma^{--} = \{\sigma_{ij}^{--}\}_{1\le i\le m, 1\le j\le m} \tag{13}$$

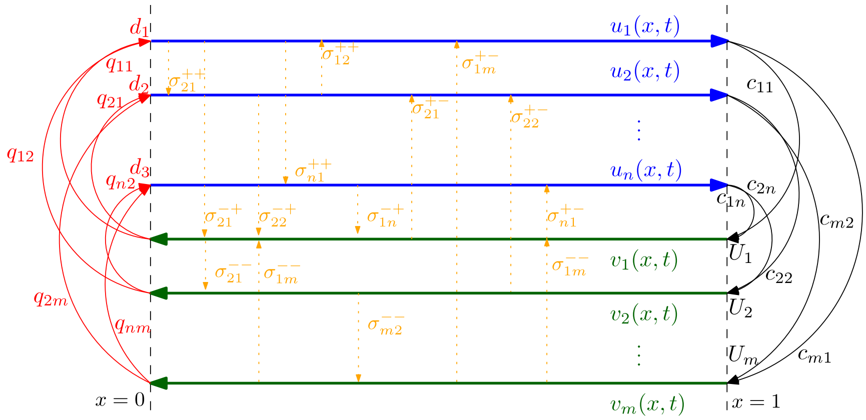

Fig. 1. System structure of u_i (blue) and v (green) with internal couplings (orange), boundary conditions at $x = 0$ (red) and boundary conditions at $x = 1$ (black). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Source: The idea for this figure is taken from Di Meglio et al. (2013).

and boundary parameters at $x = 1$ are given as

$$C_1 = \{c_{ij}\}_{1\le i\le m, 1\le j\le n}. \tag{14}$$

Furthermore, the unknown boundary parameters at $x = 0$ are

$$Q_0 = \{q_{ij}\}_{1\le i\le n, 1\le j\le m} = \begin{bmatrix} q_1 & q_2 & \cdots & q_m \end{bmatrix} \tag{15}$$

$$d = \begin{bmatrix} d_1 & \cdots & d_n \end{bmatrix}^T. \tag{16}$$

The initial conditions satisfy, for $i = 1 \ldots n, j = 1 \ldots m$

$$u_i^0, \qquad v_j^0 \in L_2([0, 1]). \tag{17}$$

The term $U(t)$ in (4) can be considered as a boundary control input, although closed-loop control is not investigated in the present work. A schematic of the structure of the system is depicted in Fig. 1.

We shall investigate the following estimation and identification problems;

Problem 1. Estimate the states of the system (1)–(4) and unknown parameters Q_0 and d in the boundary condition (3) assuming the only available measurements are

$$y(t) = u(1, t) \tag{18}$$

$$\vartheta(t) = A_0 u(0, t) + B_0 v(0, t) \tag{19}$$

where $A_0 \in \mathbb{R}^{m\times n}$ and $B_0 \in \mathbb{R}^{m\times m}$ are assumed to be matrices with known coefficients.

Problem 2. Estimate the unknown parameters Q_0 and d in the boundary condition (3) assuming the only available measurement is

$$y(t) = u(1, t). \tag{20}$$

Remark 3. While the method extends to spatially varying coefficients in (1)–(2), we consider here constant coefficients for the sake of readability.

Remark 4. We assume

$$\sigma_{ii}^{--} = 0, \quad \text{for } i = 1 \ldots m, \tag{21}$$

that is, there are no internal source terms for the v's. Such source terms can be removed by a transformation, yielding spatially varying coefficients. This is not an issue, however, in light of Remark 3.

3. Sensing at both boundaries

3.1. Problem statement

This section deals with the first stated estimation problem in which sensing at both boundaries is required, namely, Problem 1. The proposed estimation algorithm involves designing a set of filters which allows to express the system states u and v of (1)–(4) as static, linear combinations of the filters and the unknown parameters Q_0 and d. Any standard adaptive update law can then be used to estimate the unknown parameters. A similar procedure was done for the $n + 1$ case in Anfinsen, Diagne et al. (2016).

3.2. Filter design

We introduce the input filters

$$a_t(x, t) + \Lambda^+ a_x(x, t) = \Sigma^{++} a(x, t) + \Sigma^{+-} b(x, t) + K_1(x)(y(t) - a(1, t)) \tag{22}$$

$$b_t(x, t) - \Lambda^- b_x(x, t) = \Sigma^{-+} a(x, t) + \Sigma^{--} b(x, t) + K_2(x)(y(t) - a(1, t)) \tag{23}$$

with boundary conditions

$$a(0, t) = 0 \tag{24}$$

$$b(1, t) = C_1 y(t) + U(t) \tag{25}$$

where

$$a(x, t) = \begin{bmatrix} a_1(x, t) & \cdots & a_n(x, t) \end{bmatrix}^T \tag{26}$$

$$b(x, t) = \begin{bmatrix} b_1(x, t) & \cdots & b_m(x, t) \end{bmatrix}^T \tag{27}$$

and the injection gains $K_1(x)$ and $K_2(x)$ are to be designed later. These filters model how the input signal, namely $C_1 y(t) + U(t)$ affects the system states u and v.

Furthermore, we design parameter filters that are used to model how the parameters Q_0 and d influence the system states u and v. We define

$$P_t(x, t) + \Lambda^+ P_x(x, t) = \Sigma^{++} P(x, t) + \Sigma^{+-} R(x, t) - K_1(x) P(1, t) \tag{28}$$

$$R_t(x, t) - \Lambda^- R_x(x, t) = \Sigma^{-+} P(x, t) + \Sigma^{--} R(x, t) - K_2(x) P(1, t) \tag{29}$$

with boundary conditions

$$P(0, t) = \vartheta^T(t) \otimes I_{n\times n} \tag{30}$$

$$R(1, t) = 0 \tag{31}$$

where $\otimes$ denotes the Kronecker product, and

$$P(x,t) = \{p_{ij}(x,t)\}_{1\le i\le n, 1\le j\le mn} \tag{32}$$

$$R(x,t) = \{r_{ij}(x,t)\}_{1\le i\le m, 1\le j\le mn}. \tag{33}$$

Lastly, we define

$$W_t(x,t) + \Lambda^+ W_x(x,t) = \Sigma^{++}W(x,t) + \Sigma^{+-}Z(x,t) - K_1(x)W(1,t) \tag{34}$$

$$Z_t(x,t) - \Lambda^- Z(x,t) = \Sigma^{-+}W(x,t) + \Sigma^{--}Z(x,t) - K_2(x)W(1,t) \tag{35}$$

with boundary conditions

$$W(0,t) = I_{n\times n} \tag{36}$$

$$Z(1,t) = 0 \tag{37}$$

where

$$W(x,t) = \{w_{ij}(x,t)\}_{1\le i\le n, 1\le j\le n} \tag{38}$$

$$Z(x,t) = \{z_{ij}(x,t)\}_{1\le i\le m, 1\le j\le n}. \tag{39}$$

3.3. Relationship to the system states

Based on the filters introduced above, we derive static relationships to the system states. From inserting the boundary condition (3) into the measurement (19)

$$\vartheta(t) = (A_0 Q_0 + B_0)v(0,t) + A_0 d, \tag{40}$$

it is evident that we cannot estimate the unknown parameters Q_0 and d directly. Instead, we estimate the following parameters

$$\Theta = \begin{bmatrix}\theta_1 & \theta_2 & \cdots & \theta_m\end{bmatrix}, \qquad \kappa \tag{41}$$

which are related to the original parameters Q_0 and d through

$$\Theta(A_0 Q_0 + B_0) = Q_0 \tag{42}$$

$$(I - \Theta A_0)d = \kappa. \tag{43}$$

Consider the relations

$$u(x,t) = a(x,t) + P(x,t)\theta + W(x,t)\kappa + e(x,t) \tag{44}$$

$$v(x,t) = b(x,t) + R(x,t)\theta + Z(x,t)\kappa + \epsilon(x,t) \tag{45}$$

where the vector θ contains all the elements of the matrix Θ, stacked column-wise

$$\theta := \begin{bmatrix}\theta_1^T & \theta_2^T & \cdots & \theta_m^T\end{bmatrix}^T. \tag{46}$$

The following lemma holds.

Lemma 5. *The error terms e and ϵ in (44)–(45) have the following dynamics*

$$e_t(x,t) + \Lambda^+ e_x(x,t) = \Sigma^{++}e(x,t) + \Sigma^{+-}\epsilon(x,t) - K_1(x)e(1,t) \tag{47}$$

$$\epsilon_t(x,t) - \Lambda^- \epsilon_x(x,t) = \Sigma^{-+}e(x,t) + \Sigma^{--}\epsilon(x,t) - K_2(x)e(1,t) \tag{48}$$

with boundary conditions

$$e(0,t) = 0 \tag{49}$$

$$\epsilon(1,t) = 0. \tag{50}$$

Proof. The dynamics (47)–(48) follows directly from differentiating (44)–(45) with respect to time and inserting (1)–(2), (22)–(23), (28)–(29) and (34)–(35). From (44), we find

$$e(0,t) = u(0,t) - a(0,t) - P(0,t)\theta - W(0,t)\kappa = Q_0 v(0,t) + d - \Theta\vartheta(t) - I_{n\times n}\kappa. \tag{51}$$

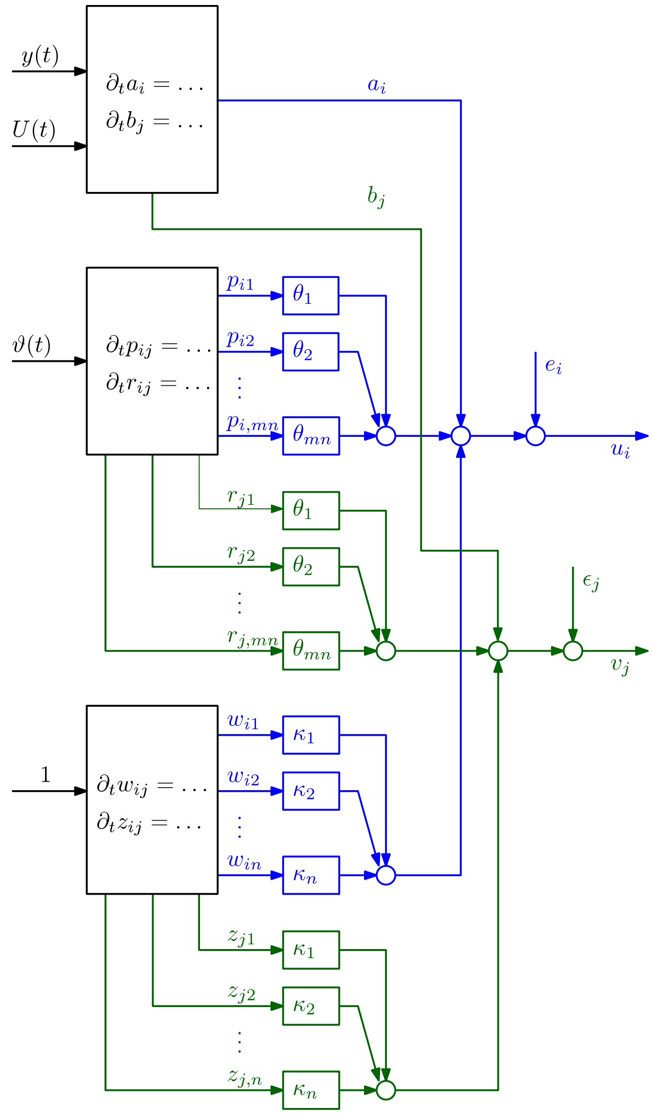

Fig. 2. Structure of the filter system, and their connection to the system states $u_i(x,t)$ (blue) and $v_j(x,t)$ (green). Only one of each state is displayed, and arguments are omitted to ease readability. This figure was inspired by a similar figure in Anfinsen, Diagne et al. (2016). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Inserting (40), we obtain

$$e(0,t) = (Q_0 - \Theta(A_0 Q_0 + B_0))\,v(0,t) + (I_{n\times n} - \Theta A_0)\,d - \kappa. \tag{52}$$

Using (42)–(43), we find (49). Lastly, from (45), we find

$$\epsilon(1,t) = v(1,t) - b(1,t) - R(1,t)\theta - Z(1,t)\kappa = C_1 u(1,t) + U(t) - C_1 y(t) - U(t) \tag{53}$$

which yields (50) when inserting (18). □

Provided the terms (e,ϵ) go to zero, relations (44)–(45) are static representations of the system states u, v from the filters and unknown parameters θ and κ, which are unique combinations of the boundary parameters Q_0 and d. A schematic of this representation is depicted in Fig. 2.

3.4. Backstepping and target system

Now, using an extended version of the backstepping transformation from Bin and Di Meglio (in press), we show that by a particular choice of the injection terms $K_1(x)$ and $K_2(x)$, the errors $e(x,t)$ and $\epsilon(x,t)$ in (44)–(45) go to zero in finite time.

Lemma 6. *The backstepping transformation*

$$\alpha(x,t) = e(x,t) - \int_x^1 M(x,\xi)\alpha(\xi,t)d\xi \tag{54}$$

$$\beta(x,t) = \epsilon(x,t) - \int_x^1 N(x,\xi)\alpha(\xi,t)d\xi \tag{55}$$

where

$$M(x,\xi) = \left\{M_{ij}(x,\xi)\right\}_{1\le i,j\le n} \tag{56}$$

$$N(x,\xi) = \left\{N_{ij}(x,\xi)\right\}_{1\le i\le n, 1\le j\le m} \tag{57}$$

are defined over the triangular domain

$$\mathcal{T} = \{(x,\xi) \mid 0 \le x \le \xi \le 1\} \tag{58}$$

and satisfy the PDEs in Appendix A.1, transforms the error system (47)–(50) into the target system

$$\alpha_t(x,t) + \Lambda^+\alpha_x(x,t) = \bar{\Sigma}^{++}\alpha(x,t) + \Sigma^{+-}\beta(x,t) - \int_x^1 D^+(x,\xi)\beta(\xi,t)d\xi \tag{59}$$

$$\beta_t(x,t) - \Lambda^-\beta_x(x,t) = \Sigma^{--}\beta(x,t) - \int_x^1 D^-(x,\xi)\beta(\xi,t)d\xi \tag{60}$$

with boundary conditions

$$\alpha(0,t) = \int_0^1 H(\xi)\alpha(\xi,t)d\xi \tag{61}$$

$$\beta(1,t) = 0 \tag{62}$$

where the injection gains are set to

$$K_1(x) = M(x,1)\Lambda^+ \tag{63}$$

$$K_2(x) = N(x,1)\Lambda^+, \tag{64}$$

the coefficients $D^+(x,\xi)$ and $D^-(x,\xi)$ are

$$D^+(x,\xi) = M(x,\xi)\Sigma^{+-} - \int_\xi^x M(x,\eta)D^+(\eta,\xi)d\eta \tag{65}$$

$$D^-(x,\xi) = N(x,\xi)\Sigma^{+-} - \int_\xi^x N(x,\eta)D^+(\eta,\xi)d\eta \tag{66}$$

with $\bar{\Sigma}^{++}$ defined as a diagonal matrix

$$\bar{\Sigma}^{++} = diag\{\sigma_{11}^{++}, \sigma_{22}^{++}, \dots, \sigma_{nn}^{++}\} \tag{67}$$

and $H(x) = \left\{h_{ij}(x)\right\}_{1\le i,j\le n}$ as a strict lower triangular matrix with components

$$h_{ij}(x) = \begin{cases} -M_{ij}(0,\xi) & \text{for } 1 \le j < i \le n \\ 0 & \text{otherwise.} \end{cases} \tag{68}$$

Proof. Differentiating (54) with respect to time, we obtain

$$e_t(x,t) = \alpha_t(x,t) + \int_x^1 M(x,\xi)\alpha_t(\xi,t)d\xi. \tag{69}$$

Inserting the α-dynamics (59), using integration by parts and changing the order of integration in the double integral terms, we derive

$$\begin{aligned} e_t(x,t) &= \alpha_t(x,t) - M(x,1)\Lambda^+\alpha(1,t) + M(x,x)\Lambda^+\alpha(x,t) \\ &+ \int_x^1 M_\xi(x,\xi)\Lambda^+\alpha(\xi,t)d\xi \\ &+ \int_x^1 M(x,\xi)\bar{\Sigma}^{++}\alpha(\xi,t)d\xi \\ &+ \int_x^1 M(x,\xi)\Sigma^{+-}\beta(\xi,t)d\xi \\ &- \int_x^1\int_\xi^x M(x,\eta)D^+(\eta,\xi)d\eta\beta(\xi,t)d\xi. \end{aligned} \tag{70}$$

Next, differentiating (54) with respect to space using Leibniz's rule, we arrive at

$$e_x(x,t) = \alpha_x(x,t) - M(x,x)\alpha(x,t) + \int_x^1 M_x(x,\xi)\alpha(\alpha,t)d\xi. \tag{71}$$

Substituting (70)–(71) into (47) and using (54)–(55), we obtain

$$\begin{aligned} &\tilde{\alpha}_t(x,t) + \Lambda^+\tilde{\alpha}_x(x,t) \\ &= \Sigma^{+-}\tilde{\beta}(x,t) \\ &- \int_x^1 \big[M_\xi(x,\xi)\Lambda^+ + M(x,\xi)\bar{\Sigma}^{++} + \Lambda^+M_x(x,\xi) \\ &- \Sigma^{++}M(x,\xi) - \Sigma^{+-}N(x,\xi)\big]\tilde{\alpha}(\xi,t)d\xi \\ &- \left[K_1(x) - M(x,1)\Lambda^+\right]\tilde{\alpha}(1,t) - \int_x^1 \Big[M(x,\xi)\Sigma^{+-} \\ &- \int_\xi^x M(x,\eta)D^+(\eta,\xi)d\eta\Big]\tilde{\beta}(\xi,t)d\xi \\ &+ \left[\Lambda^+M(x,x) - M(x,x)\Lambda^+ + \Sigma^{++}\right]\tilde{\alpha}(x,t). \end{aligned} \tag{72}$$

Using (63), (65), (A.1) and (A.3), Eq. (59) is deduced. Similar derivations for (55), using (48) and (59) yield

$$\begin{aligned} &\tilde{\beta}_t(x,t) - \Lambda^-\tilde{\beta}_x(x,t) \\ &= \Sigma^{--}\tilde{\beta}(x,t) \\ &+ \int_x^1 \big[\Lambda^-N_x(x,\xi) - N_\xi(x,\xi)\Lambda^+ + \Sigma^{-+}M(x,\xi) \\ &+ \Sigma^{--}N(x,\xi) - N(x,\xi)\bar{\Sigma}^{++}\big]\tilde{\alpha}(\xi,t)d\xi \\ &- \int_x^1 \Big[N(x,\xi)\Sigma^{+-} - \int_\xi^x N(x,\eta)D^+(\eta,\xi)d\eta\Big]\tilde{\beta}(\xi,t)d\xi \\ &- \left[\Lambda^-N(x,x) + N(x,x)\Lambda^+ - \Sigma^{-+}\right]\tilde{\alpha}(x,t) \\ &+ \left[N(x,1)\Lambda^+ - K_2(x)\right]. \end{aligned} \tag{73}$$

Finally, using (A.2), (A.4), (64) and (66), one can derive (60) and substituting (54) into (49), from the definition of $H(\xi)$ stated in (68), we derive the first boundary condition (61). The last boundary condition (62) follows directly from evaluating (55) at $x = 1$ and using (50). □

3.5. Stability of the target system

Recall that since the backstepping transformation in Lemma 6 is invertible, the stability properties of the error system (47)–(50) and the target system (59)–(62) are equivalent. Stabilizing (59)–(62) will thus result in stabilizing (47)–(50).

Lemma 7. *The target system (59)–(62) tends to zero in a finite time given by*

$$t_F = \mu_1^{-1} + \sum_{k=1}^n \lambda_k^{-1}. \tag{74}$$

Proof. We argue in a similar way as in Hu et al. (2016). The components of Eq. (60), for $i = 1 \dots m$ are given by

$$\partial_t \beta_i(x,t) - \mu_i \partial_x \beta_i(x,t) = \sum_{k=1}^{m} \sigma_{ik}^{--} \beta_k(x,t) - \sum_{k=1}^{m} \int_x^1 d_{ik}^{-}(x,\xi)\beta_k(\xi,t)d\xi \tag{75}$$

with boundary conditions

$$\beta_i(1,t) = 0, \tag{76}$$

where $d_{ij}(x,\xi)$ are the components of $D^-(x,\xi)$. Equations (75)–(76) constitute m transport equations with a zero boundary condition that will all be zero when the slowest one, β_1 has propagated its boundary condition to $x = 1$. Thus, after

$$t = t_0 = \mu_1^{-1} \tag{77}$$

one obtains

$$\beta_i(x,t) \equiv 0, \quad i = 1 \dots m. \tag{78}$$

Hence, for $t > t_0$, the target system (59)–(62) is reduced to

$$\alpha_t(x,t) + \Lambda^+ \alpha_x(x,t) = \bar{\Sigma}^{++}\alpha(x,t) \tag{79}$$

$$\alpha(0,t) = \int_0^1 H(\xi)\alpha(\xi,t)d\xi \tag{80}$$

or to the following, when written out in its components, for $i = 1 \dots n$

$$\partial_t \alpha_i(x,t) + \lambda_i \partial_x \alpha_i(x,t) = \sigma_{ii}^{++}\alpha_i(x,t) \tag{81}$$

$$\alpha_i(0,t) = \sum_{k=1}^{i-1} h_{ik}(\xi)\alpha_k(\xi,t)d\xi \tag{82}$$

which is a cascade structure. For $i = 1$ we have

$$\partial_t \alpha_1(x,t) + \lambda_1 \partial_x \alpha_1(x,t) = \sigma_{11}^{++}\alpha_1(x,t) \tag{83}$$

$$\alpha_1(0,t) = 0, \tag{84}$$

which is exactly equal to zero after a total time of $t = t_0 + \lambda_1^{-1}$. Consequently, the second component of (79)–(80) is then

$$\partial_t \alpha_2(x,t) + \lambda_2 \partial_x \alpha_2(x,t) = \sigma_{22}^{++}\alpha_2(x,t) \tag{85}$$

$$\alpha_2(0,t) = 0 \tag{86}$$

which is zero after a total time of $t = t_0 + \lambda_1^{-1} + \lambda_2^{-1}$. Considering the same procedure for $i = 3 \dots n$, we deduce by induction that the total system is zero after the time given by (74). □

3.6. Update law and state estimates

From the static form (44)–(45) with error terms converging to zero, one can follow the same approach as in Anfinsen, Diagne et al. (2016) and use standard gradient and least squares update laws to estimate the parameters in θ and κ. From (44), we deduce the following relation

$$\begin{aligned} e(1,t) &= u(1,t) - a(1,t) - P(1,t)\theta - W(1,t)\kappa \\ &= y(t) - a(1,t) - P(1,t)\theta - W(1,t)\kappa. \end{aligned} \tag{87}$$

Next, we define the following vector

$$\phi(t) := y(t) - a(1,t) \tag{88}$$

and the matrix

$$\Phi(t) = \begin{bmatrix} P(1,t) & W(1,t) \end{bmatrix}. \tag{89}$$

Then, Eq. (87) can be written

$$e(1,t) = \phi(t) - \Phi(t)\nu, \tag{90}$$

where

$$\nu = \begin{bmatrix} \theta^T & \kappa^T \end{bmatrix}^T. \tag{91}$$

Theorem 8. *Consider the system (1)–(4) with filters (22)–(25), (28)–(31) and (34)–(37) and with injection gains given by (63)–(64). Then the following normalized update law*

$$\dot{\hat{\nu}}(t) = s(t)\Gamma \frac{\Phi^T(t)(\phi(t) - \Phi(t)\hat{\nu}(t))}{1 + \|\Phi^T(t)\Phi(t)\|^2} \tag{92}$$

where $\Gamma > 0$ is a gain, and

$$s(t) = \begin{cases} 1 & \text{if } t > t_F \\ 0 & \text{otherwise} \end{cases} \tag{93}$$

with t_F given in (74), ensures that $\tilde{\nu} = \hat{\nu} - \nu \in \mathcal{L}_\infty$. Moreover, if $\Phi(t)$ and $\dot{\Phi}(t)$ are bounded and $\Phi^T(t)$ is persistently exciting (PE), that is; there exist positive constants T_0, c_0, c_1 so that

$$c_0 \mathcal{I} \le \frac{1}{T_0}\int_t^{t+T_0} \Phi^T(\tau)\Phi(\tau)d\tau \le c_1 \mathcal{I} \tag{94}$$

where

$$\mathcal{I} = I_{(m+1)n \times (m+1)n} \tag{95}$$

for all $t \ge 0$, then $\hat{\nu} \to \nu$ exponentially fast.

Proof. We construct a "prediction error" as follows

$$\hat{e}(1,t) = \phi(t) - \Phi(t)\hat{\nu}(t). \tag{96}$$

Next, consider the Lyapunov function candidate

$$V(t) = \frac{1}{2}\tilde{\nu}^T(t)\Gamma^{-1}\tilde{\nu}(t) \tag{97}$$

where $\tilde{\nu}(t) := \hat{\nu}(t) - \nu$. The time derivative of (97) is written as

$$\dot{V}(t) = s(t)\tilde{\nu}^T(t)\frac{\Phi^T(t)\hat{e}(1,t)}{1 + \|\Phi^T(t)\Phi(t)\|^2}. \tag{98}$$

Noticing that

$$\hat{e}(1,t) = \phi(t) - \Phi(t)\hat{\nu}(t) = e(1,t) - \Phi(t)\tilde{\nu}(t), \tag{99}$$

we arrive at

$$\dot{V}(t) = s(t)\frac{\tilde{\nu}^T(t)\Phi^T(t)e(1,t)}{1 + \|\Phi^T(t)\Phi(t)\|^2} - s(t)\frac{|\Phi(t)\tilde{\nu}(t)|^2}{1 + \|\Phi^T(t)\Phi(t)\|^2}. \tag{100}$$

From (93), the first term of Eq. (100) is zero since for $t \le t_F$, we have $s(t) = 0$, and for $t > t_F$, we have $e(1,t) = 0$ and $s(t) = 1$, hence for $t > t_F$

$$\dot{V}(t) = -\frac{|\Phi(t)\tilde{\nu}(t)|^2}{1 + \|\Phi^T(t)\Phi(t)\|^2}. \tag{101}$$

The above equality shows that V is non-increasing and hence bounded, which in turn implies that $\tilde{\nu}$ is bounded.

We recall that in Section 3.5, the convergence of e and ϵ to zero in a finite time and independently of the update law of Theorem 8 is stated. Therefore, the static form of the measurements in (90) eventually reaches the form $\phi(t) = \Phi(t)\nu$. The latter part of Theorem 8 then follows from part (iii) of Theorem 4.3.2 in Ioannou and Sun (1995). □

Remark 9. The requirement of having $\Phi(t)$ and $\dot{\Phi}(t)$ bounded is ensured if the plant is stable.

Let $\hat{\Theta}(t)$ be the matrix formed by stacking the components of $\hat{\theta}(t)$ column-wise, that is

$$\hat{\Theta}(t) = \begin{bmatrix}\hat{\theta}_1(t) & \hat{\theta}_2(t) & \cdots & \hat{\theta}_m(t)\end{bmatrix} \tag{102}$$

where

$$\hat{\theta}(t) = \begin{bmatrix}\hat{\theta}_1^T(t) & \hat{\theta}_2^T(t) & \cdots & \hat{\theta}_m^T(t)\end{bmatrix}^T \tag{103}$$

and

$$\hat{\nu}(t) = \begin{bmatrix}\hat{\theta}^T(t) & \hat{\kappa}^T(t)\end{bmatrix}^T \tag{104}$$

are the estimates for the parameters derived from the update law of Theorem 8, namely, (92). One should notice that estimates for the parameters Q_0 and d can be obtained from solving (42)–(43) with respect to Q_0 and d, and replace Θ and κ with their respective estimates

$$\hat{Q}_0(t) = (I - \hat{\Theta}(t)A_0)^{-1}\hat{\Theta}(t)B_0 \tag{105}$$

$$\hat{d}(t) = (I - \hat{\Theta}(t)A_0)^{-1}\hat{\kappa}(t), \tag{106}$$

whenever $I - \hat{\Theta}(t)A_0$ is nonsingular (well-conditioned). Finally, estimates for the states of the system can be derived from the relationship (44)–(45) and the parameter estimates generated from the update law of Theorem 8, namely, (92). That is

$$\hat{u}(x,t) = a(x,t) + P(x,t)\hat{\theta}(t) + W(x,t)\hat{\kappa}(t) \tag{107}$$

$$\hat{v}(x,t) = b(x,t) + R(x,t)\hat{\theta}(t) + Z(x,t)\hat{\kappa}(t). \tag{108}$$

Remark 10. The matrix $(I - \hat{\Theta}(t)A_0)$ may very well be singular or close to singular, particularly in view of the fact that we have not ruled out that $(I - \Theta A_0)$ may be singular. This is a question of uniqueness of the parameterizations (42) and (43). From (42), it is observed that Θ uniquely parametrizes Q_0 provided

$$\det(A_0Q_0 + B_0) \neq 0. \tag{109}$$

If (109) holds, then Lemma 13 in Appendix B states that the matrix $I - \Theta A_0$ is invertible if and only if

$$\det(B_0) \neq 0. \tag{110}$$

Requirement (109) can also be interpreted as a question of observability, since a singular matrix $A_0Q_0 + B_0$ renders some of the states in $v(0,t)$ in (40), unobservable. The requirement (110) is a question of identifiability of the parameters Q_0.[1] If the matrix $I - \hat{\Theta}(t)A_0$ is singular or close to singular, one can either conclude that the conditions (109) and (110) do not hold for the system, or that one lacks persistence of excitation.

4. Sensing anti-collocated with the uncertain parameters

4.1. Problem statement

In the present section, we consider the problem of estimating the unknown boundary parameters Q_0 and d as in Problem 1, with one important distinction: the only available measurement is now assumed to be the one anti-collocated with the uncertainties, i.e., the measurement (18). This problem, whose mathematical formulation is stated as Problem 2, is often more relevant than Problem 1 in practice. One example is oil well drilling, where it is of interest to estimate reservoir pressure at the bottom of the well from sensing equipment mounted topside. Such an estimation problem was recently solved for $n+1$ hyperbolic systems in Anfinsen, Di Meglio et al. (2016), using swapping design and time-delaying measurements. We propose a similar method to the one employed to solve Problem 1 and will reuse some of the filters presented in Section 3.

[1] This requirement is most easily demonstrated using the 1-D case. It corresponds to the problem of estimating the parameter q from the measurements g and h, given from $g(t) = qf(t)$, $h(t) = (q+b)f(t)$. The parameter q is not identifiable for $b = 0$.

4.2. Filter design

First off, we store the past values of the measurement $y(t)$ by introducing a new filter defined as the following transport equation

$$\mathcal{Y}_t(x,t) + \lambda\mathcal{Y}_x(x,t) = 0 \tag{111}$$

with boundary condition

$$\mathcal{Y}(0,t) = y(t) \tag{112}$$

where

$$\mathcal{Y}(t) = \begin{bmatrix}\mathcal{Y}_1(t) & \mathcal{Y}_2(t) & \cdots & \mathcal{Y}_n(t)\end{bmatrix}^T \tag{113}$$

and

$$\lambda := \min_i \lambda_i = \lambda_1 \tag{114}$$

provides past values of $y(t)$, since

$$y(t - \lambda^{-1}x) = \mathcal{Y}(x,t). \tag{115}$$

We also construct filters generating time-delayed signal values of the input filters (22)–(23). Introducing

$$\mathcal{A}_t(x,\xi,t) + \lambda\mathcal{A}_x(x,\xi,t) = 0 \tag{116}$$

$$\mathcal{B}_t(x,\xi,t) + \lambda\mathcal{B}_x(x,\xi,t) = 0 \tag{117}$$

with boundary conditions

$$\mathcal{A}(0,\xi,t) = a(\xi,t) \tag{118}$$

$$\mathcal{B}(0,\xi,t) = b(\xi,t) \tag{119}$$

where

$$\mathcal{A}(x,\xi,t) = \begin{bmatrix}\mathcal{A}_1(x,\xi,t) & \cdots & \mathcal{A}_n(x,\xi,t)\end{bmatrix}^T, \tag{120}$$

$$\mathcal{B}(x,\xi,t) = \begin{bmatrix}\mathcal{B}_1(x,\xi,t) & \cdots & \mathcal{B}_m(x,\xi,t)\end{bmatrix}^T. \tag{121}$$

The filters (116)–(119) ensure the availability of the previous values of a and b, since solving these PDEs by the method of characteristics enables expressing their solutions as

$$a(\xi, t - \lambda^{-1}x) = \mathcal{A}(x,\xi,t) \tag{122}$$

$$b(\xi, t - \lambda^{-1}x) = \mathcal{B}(x,\xi,t). \tag{123}$$

Lastly, we slightly modify the $P - R$ filters of (28)–(31) and define the following filters

$$\check{P}_t(x,t) + \Lambda^+\check{P}_x(x,t) = \Sigma^{++}\check{P}(x,t) + \Sigma^{+-}\check{R}(x,t) - K_1(x)\check{P}(1,t) \tag{124}$$

$$\check{R}_t(x,t) - \Lambda^-\check{R}_x(x,t) = \Sigma^{-+}\check{P}(x,t) + \Sigma^{--}\check{R}(x,t) - K_2(x)\check{P}(1,t) \tag{125}$$

with boundary conditions

$$\check{P}(0,t) = \bar{p}^T(t) \otimes I_{n\times n} \tag{126}$$

$$\check{R}(1,t) = 0 \tag{127}$$

where

$$\bar{p}(t) = v(0, t - \lambda^{-1}). \tag{128}$$

Recall that $v(0,t)$ is not measured, so (128) cannot be implemented. In Section 4.3, we derive an alternative way of computing $\bar{p}(t)$ using measured signals, only.

4.3. Computation of the signal $\bar{p}(t)$

The filters designed in Section 4.2 require Eq. (128), but the boundary value $v(0,t)$ is not measured. It turns out, however, that $v(0, t-\lambda^{-1})$ can be expressed in closed form using available measurements, only.

Lemma 11. *Consider the system (1)–(4) and the filters (111)–(112) and (116)–(119). Let the vector of signals $\bar{p}(t) = \{\bar{p}_i(t)\}$, $i = 1\dots n$ have elements*

$$\begin{aligned}\bar{p}_i(t) &= \mathcal{B}_i(1,0,t) + \sum_{k=1}^{n}\int_0^1 N_{ik}(0,\xi)\exp\left(-\frac{\sigma_{kk}}{\lambda_k}(1-\xi)\right)\\ &\quad\times\left[\mathcal{Y}_k(1-\lambda_k^{-1}\lambda(1-\xi),t)\right.\\ &\quad\left.-\mathcal{A}_k(1-\lambda_k^{-1}\lambda(1-\xi),1,t)\right]d\xi \qquad (129)\end{aligned}$$

where N_{ij}, are the components of N of the solution (M,N) of (A.1)–(A.6). Then for $t > t_F$ where t_F is given from (74), we have

$$\bar{p}(t) = v(0, t-\lambda^{-1}). \qquad (130)$$

Proof. Construct the signals

$$\tilde{u}(x,t) = u(x,t) - a(x,t) \qquad (131)$$
$$\tilde{v}(x,t) = v(x,t) - b(x,t) \qquad (132)$$

which have the following dynamics

$$\begin{aligned}\tilde{u}_t(x,t) + \Lambda^+\tilde{u}_x(x,t) &= \Sigma^{++}\tilde{u}(x,t) + \Sigma^{+-}\tilde{v}(x,t)\\ &\quad - K_1(x)\tilde{u}(1,t) \qquad (133)\\ \tilde{v}_t(x,t) + \Lambda^-\tilde{v}_x(x,t) &= \Sigma^{-+}\tilde{u}(x,t) + \Sigma^{--}\tilde{v}(x,t)\\ &\quad - K_2(x)\tilde{u}(1,t) \qquad (134)\end{aligned}$$

with the boundary conditions

$$\tilde{u}(0,t) = Q_0 v(0,t) + d \qquad (135)$$
$$\tilde{v}(1,t) = 0. \qquad (136)$$

Similarly to the backstepping transformation of Lemma 6, we introduce the following backstepping transformation

$$\check{\alpha}(x,t) = \tilde{u}(x,t) - \int_x^1 M(x,\xi)\check{\alpha}(\xi,t)d\xi \qquad (137)$$
$$\check{\beta}(x,t) = \tilde{v}(x,t) - \int_x^1 N(x,\xi)\check{\alpha}(\xi,t)d\xi, \qquad (138)$$

which maps the system (133)–(136) to the following target system

$$\begin{aligned}\check{\alpha}_t(x,t) + \Lambda^+\check{\alpha}_x(x,t) &= \bar{\Sigma}^{++}\check{\alpha}(x,t) + \Sigma^{+-}\check{\beta}(x,t)\\ &\quad - \int_x^1 D^+(x,\xi)\check{\beta}(\xi,t)d\xi \qquad (139)\\ \check{\beta}_t(x,t) - \Lambda^-\check{\beta}_x(x,t) &= \Sigma^{--}\check{\beta}(x,t)\\ &\quad - \int_x^1 D^-(x,\xi)\check{\beta}(\xi,t)d\xi \qquad (140)\end{aligned}$$

with boundary conditions

$$\check{\alpha}(0,t) = \int_0^1 H(\xi)\check{\alpha}(\xi,t)d\xi + Q_0 v(0,t) + d \qquad (141)$$
$$\check{\beta}(1,t) = 0 \qquad (142)$$

where $\bar{\Sigma}^{++}$, $M(x,\xi)$, $N(x,\xi)$, $D^+(x,\xi)$, $D^-(x,\xi)$ and $H(\xi)$ are as in Section 3.4. The derivations are the same as in the proof of Lemma 6, and are therefore omitted.

Now, similarly to Lemma 7, we will for $t = t_0 = \mu_1^{-1}$, have $\check{\beta} \equiv 0$, and the target error system (139)–(142) is therefore reduced to

$$\check{\alpha}_t(x,t) + \Lambda^+\check{\alpha}_x(x,t) = \bar{\Sigma}\check{\alpha}(x,t) \qquad (143)$$
$$\begin{aligned}\check{\alpha}(0,t) &= \int_0^1 H(\xi)\check{\alpha}(\xi,t)d\xi\\ &\quad + Q_0 v(0,t) + d \qquad (144)\end{aligned}$$

or when written out in its components, yields, for $i = 1\dots n$

$$\partial_t\check{\alpha}_i(x,t) + \lambda_i\partial_x\check{\alpha}_i(x,t) = \sigma_{ii}\check{\alpha}_i(x,t) \qquad (145)$$
$$\check{\alpha}_i(0,t) = \sum_{j=1}^{i-1}\int_0^1 h_{ij}(\xi)\check{\alpha}_j(\xi,t)d\xi + \sum_{k=1}^{m} q_{ik}v_k(0,t) + d_i. \qquad (146)$$

The solution of (145) is

$$\check{\alpha}_i(x,t) = \exp\left(\frac{\sigma_{ii}}{\lambda_i}x\right)\check{\alpha}_i(0, t-\lambda_i^{-1}x). \qquad (147)$$

Particularly,

$$\check{\alpha}_i(1,t) = u_i(1,t) - a_i(1,t) = y_i(t) - a_i(1,t), \qquad (148)$$

which means that $\check{\alpha}_i(x,t)$ can be expressed using $y_i(t)$ and $a_i(1,t)$ as follows

$$\begin{aligned}\check{\alpha}_i(x,t) &= \exp\left(-\frac{\sigma_{ii}}{\lambda_i}(1-x)\right)\\ &\quad\times\left[y_i(t+\lambda_i^{-1}(1-x)) - a_i(1, t+\lambda_i^{-1}(1-x))\right]. \qquad (149)\end{aligned}$$

Again using the fact that $\check{\beta} \equiv 0$ for $t > t_0 = \mu_1^{-1}$, we specifically have that $\check{\beta}(0,t) = 0$, and from (138) we find

$$v(0,t) = b(0,t) + \int_0^1 N(0,\xi)\check{\alpha}(\xi,t)d\xi. \qquad (150)$$

Next, with the help of (149), we may write them component wise as

$$\begin{aligned}v_i(0,t) &= b_i(0,t) + \sum_{k=1}^{n}\int_0^1 N_{ik}(0,\xi)\exp\left(-\frac{\sigma_{kk}}{\lambda_k}(1-\xi)\right)\\ &\quad\times\left[y_k(t+\lambda_k^{-1}(1-\xi)) - a_k(1,t+\lambda_k^{-1}(1-\xi))\right]d\xi \qquad (151)\end{aligned}$$

for $1 \le i \le m$, where $N_{ij}(0,\xi)$ are the components of $N(0,\xi)$. This yields a way to compute $v(0,t)$ from the measurements and the signal $b(0,t)$. However, it depends upon future values of $y(t)$ and $a(1,t)$. What may be computed, however, are the time-delayed versions of $v_i(0,t)$

$$\begin{aligned}v_i(0,t-\lambda^{-1}) &= \mathcal{B}_i(1,0,t)\\ &\quad + \sum_{k=1}^{n}\int_0^1 N_{ik}(0,\xi)\exp\left(-\frac{\sigma_{kk}}{\lambda_k}(1-\xi)\right)\\ &\quad\times\left[\mathcal{Y}_k(1-\lambda_k^{-1}\lambda(1-\xi),t)\right.\\ &\quad\left.-\mathcal{A}_k(1-\lambda_k^{-1}\lambda(1-\xi),1,t)\right]d\xi. \qquad (152)\end{aligned}$$

Thus, we choose the elements of $\bar{p}(t)$ in (126) as (129), which makes $\bar{p}(t)$ equal to $v(0,t-\lambda^{-1})$ for $t > t_F$, where t_F is defined in (74), and expressed using available filters and measurements only. □

4.4. Relationship to the system states and convergence

With the filters derived above, we define the following relations to the system states

$$u(x, t-\lambda^{-1}) = \mathcal{A}(1, x, t) + \check{P}(x, t)q + W(x, t)d + \check{e}(x, t) \quad (153)$$

$$v(x, t-\lambda^{-1}) = \mathcal{B}(1, x, t) + \check{R}(x, t)q + Z(x, t)d + \check{\epsilon}(x, t), \quad (154)$$

where q contains the elements of Q_0 stacked column-wise, that is

$$q = \begin{bmatrix} q_1^T & q_2^T & \cdots & q_m^T \end{bmatrix}^T. \quad (155)$$

As in Lemma 5, the error terms have the following dynamics

$$\check{e}_t(x, t) + \Lambda^+ \check{e}_x(x, t) = \Sigma^{++}\check{e}(x, t) + \Sigma^{+-}\check{\epsilon}(x, t) - K_1(x)\check{e}(1, t) \quad (156)$$

$$\check{\epsilon}_t(x, t) - \Lambda^- \check{\epsilon}_x(x, t) = \Sigma^{-+}\check{e}(x, t) + \Sigma^{--}\check{\epsilon}(x, t) - K_2(x)\check{e}(1, t) \quad (157)$$

with boundary conditions

$$\check{e}(0, t) = 0 \quad (158)$$

$$\check{\epsilon}(1, t) = 0. \quad (159)$$

The error system (156)–(159) has the exact same structure as the error system (47)–(50), which was shown in Lemma 7 in Section 3.5 to converge to zero in finite time t_F, given by (74).

4.5. Update law

Next, we derive the update law in order to estimate the unknown boundary parameters from sensing anti-collocated with the uncertain parameters. From the static relations (153), we obtain

$$\begin{aligned}\check{e}(1, t) &= u(1, t-\lambda^{-1}) - \mathcal{A}(1, x, t) - \check{P}(1, t)q - W(1, t)d \\ &= \mathcal{Y}(1, t) - \mathcal{A}(1, 1, t) - \check{P}(1, t)q - W(1, t)d. \end{aligned} \quad (160)$$

Next, we consider the following signals

$$\check{\phi}(t) = \mathcal{Y}(1, t) - \mathcal{A}(1, 1, t) \quad (161)$$

$$\check{\Phi}(t) = \begin{bmatrix} \check{P}(1, t) & W(1, t) \end{bmatrix} \quad (162)$$

and derive the following relation

$$\check{e}(1, t) = \check{\phi}(t) - \check{\Phi}(t)\nu_2 \quad (163)$$

where ν_2 is

$$\nu_2 = \begin{bmatrix} q^T & d^T \end{bmatrix}^T. \quad (164)$$

Theorem 12. *Consider the system (1)–(4) with filters (22)–(25), (116)–(119), (34)–(37) and (124)–(127) and with injection gains given by (63)–(64). Then the following normalized update law*

$$\dot{\hat{\nu}}_2(t) = s(t)\check{\Gamma}\frac{\check{\Phi}^T(t)(\check{\phi}(t) - \check{\Phi}(t)\hat{\nu}_2(t))}{1 + \|\check{\Phi}^T(t)\check{\Phi}(t)\|^2} \quad (165)$$

where $\check{\Gamma} > 0$ is a gain and $s(t)$ is as defined in (93), ensures that $\tilde{\nu}_2 = \hat{\nu}_2 - \nu_2 \in \mathcal{L}_\infty$. Moreover, if $\check{\Phi}(t)$ and $\dot{\check{\Phi}}(t)$ are bounded and $\check{\Phi}^T(t)$ is persistently exciting (PE) as defined in (94), then the system parameters converge to their true values exponentially.

Proof. The proof follows the exact same steps as the proof of Theorem 8, and is therefore omitted. □

From the static relations (153)–(154), one can generate estimates of the system states by substituting q and d with their estimates generated from the adaptation law of Theorem 12, namely, (165), thus

$$\hat{u}(x, t-\lambda^{-1}) = \mathcal{A}(1, x, t) + \check{P}(x, t)\hat{q}(t) + W(x, t)\hat{d}(t) \quad (166)$$

$$\hat{v}(x, t-\lambda^{-1}) = \mathcal{B}(1, x, t) + \check{R}(x, t)\hat{q}(t) + Z(x, t)\hat{d}(t) \quad (167)$$

where

$$\hat{\nu}_2(t) = \begin{bmatrix} \hat{q}(t) & \hat{d}(t) \end{bmatrix}, \quad (168)$$

which generate estimates of the system states from an earlier point in time which precedes the real time by a time interval that corresponds to the slowest of the transport speeds (λ^{-1}) that separate the boundary where sensing takes place from the boundary at which the uncertainty is present. More sophisticated methods will be needed to generate system states in real time.

5. Simulations

The system (1)–(4) was implemented in MATLAB for $m = n = 2$. The system transport speeds were set to

$$\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \qquad \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \quad (169)$$

with the in-domain parameters set to

$$\begin{bmatrix} \sigma_{11}^{++} & \sigma_{12}^{++} & \sigma_{11}^{+-} & \sigma_{12}^{+-} \\ \sigma_{21}^{++} & \sigma_{22}^{++} & \sigma_{21}^{+-} & \sigma_{22}^{+-} \\ \sigma_{11}^{-+} & \sigma_{12}^{-+} & \sigma_{11}^{--} & \sigma_{12}^{--} \\ \sigma_{21}^{-+} & \sigma_{22}^{-+} & \sigma_{21}^{--} & \sigma_{22}^{--} \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 0 & 2 & 2 & -2 \\ 2 & 0 & 4 & 0 \\ 4 & 2 & 0 & 2 \\ -1 & 2 & 2 & 0 \end{bmatrix} \quad (170)$$

and the boundary parameter C_1 set to

$$C_1 = \frac{1}{10}\begin{bmatrix} 1 & -2 \\ 0.5 & 0.3 \end{bmatrix}, \quad (171)$$

while the unknown parameters to be identified were set to

$$Q_0 = \frac{1}{5}\begin{bmatrix} 0 & 5 \\ 2 & -5 \end{bmatrix}, \qquad d = \frac{1}{10}\begin{bmatrix} -7 \\ 6 \end{bmatrix}. \quad (172)$$

This plant is open-loop stable. The matrices in the measurement (19) were set to

$$A_0 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \qquad B_0 = \begin{bmatrix} 0.5 & 2 \\ 1 & 1 \end{bmatrix} \quad (173)$$

which ensures that both (110) and (109) are satisfied. The initial values for the plant were set to

$$u_1(x, 0) = \sin(x), \qquad u_2(x, 0) = e^x\cos(x) \quad (174)$$

$$v_1(x, 0) = \sin(2\pi(1 - x)), \qquad v_2(x, 0) = x \quad (175)$$

to create some transients.

The initial values for the filters were in both cases all set to zero. The following was used as an input to ensure the PE properties of the regressors

$$U_1(t) = \sin(t) + \sin\left(\frac{1}{2}\sqrt{3}t\right) \quad (176)$$

$$U_2(t) = \sin(2\sqrt{2}t). \quad (177)$$

The adaptation gain was set to

$$\Gamma = 8 \cdot I_{6\times 6}, \quad (178)$$

while the backstepping kernel equations (A.1)–(A.6) were solved by successive approximations. (See e.g. the proof of existence of solution to the kernel equations in Hu et al. (2016) for details

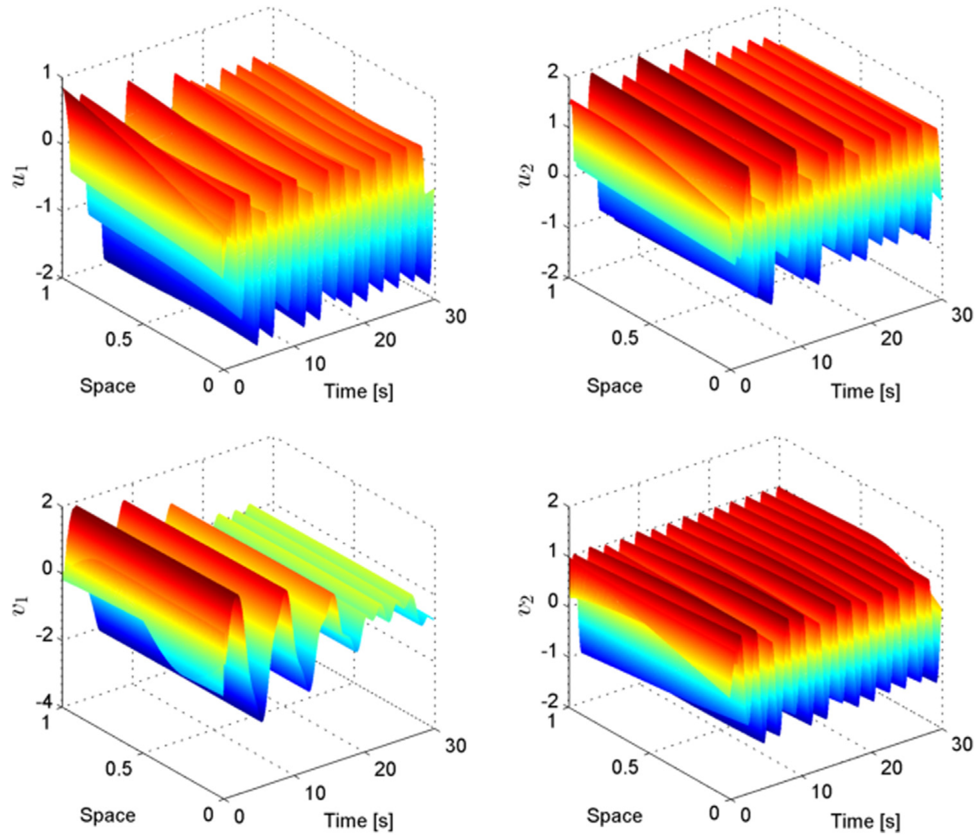

Fig. 3. System states, observer of Theorem 8.

regarding how to bring the equations to integral form, and perform successive approximations.) Since these equations are independent of time, their solution can be computed once off-line before any additional estimation takes place. The computational cost is therefore of minor concern for a real-time observer. The initial guesses of the parameters to be estimated were in both cases all set to zero.

5.1. Problem 1: Sensing at both boundaries

The adaptive observer of Theorem 8 was here implemented.

The system states are displayed in Fig. 3, while the "prediction errors" $\hat{e} = u - \hat{u}$, $\hat{\epsilon} = v - \hat{v}$ are displayed in Fig. 4. The estimated multiplicative parameters $\hat{q}_{ij}$ as computed from (105) and the estimated additive parameters $\hat{d}_i$ computed from (106) are displayed in Fig. 5. It is observed that the estimated parameters converge to their true values as predicted by the theory, with the prediction errors also converging to zero.

5.2. Problem 2: Sensing anti-collocated with the uncertainties

The system was here implemented with the adaptive observer of Theorem 12. The estimated multiplicative and additive parameters, shown in Fig. 6, converge to their true values. It is worth noticing that the estimation time is very similar to the one for the observer of Theorem 8.

6. Conclusion

We have developed two adaptive observers for estimating unknown parameters in the boundary condition of a general system of $n + m$ coupled, first-order 1-D hyperbolic PDEs. The observers use swapping design in order the express the system states as static relations between the unknown parameters and the filters. Standard gradient or least squares update laws can then be applied. Boundedness of the adaptive laws is proved. Exponential convergence of the estimated parameters is also proved in the presence of persistently exciting (PE) regressors. One observer manages to estimate the unknown boundary parameters as well as the system states from sensing at both boundaries, while the second observer enables to estimate the unknown parameters from sensing restricted to be anti-collocated with the uncertain parameters. However, the latter observer generates estimates of the system states from an earlier point in time which precedes the real time by a time interval that corresponds to the fastest of the transport speeds that separate the boundary where sensing takes place from the boundary at which the uncertainty is present. The theory was demonstrated in simulations.

The proposed adaptive observers have some limitations. The observability and identifiability of the unknown parameters for Problem 1 rely on conditions depending of the unknown parameters, and hence, cannot be checked beforehand. For Problem 2, real-time estimates of the system states are not obtained. It is also difficult (or impossible) to check conditions for persistent excitation, which is necessary for convergence of the estimates to the true values of the parameters.

Possible future extensions include relaxations of the aforementioned limitations. Deriving an observer that manages to estimate the boundary parameters and simultaneously generate real-time estimates of the system states from sensing anti-collocated with the uncertain parameters is of great interest, because it facilitates for closed loop adaptive control of the system (1)–(4) with collocated sensing and actuation.

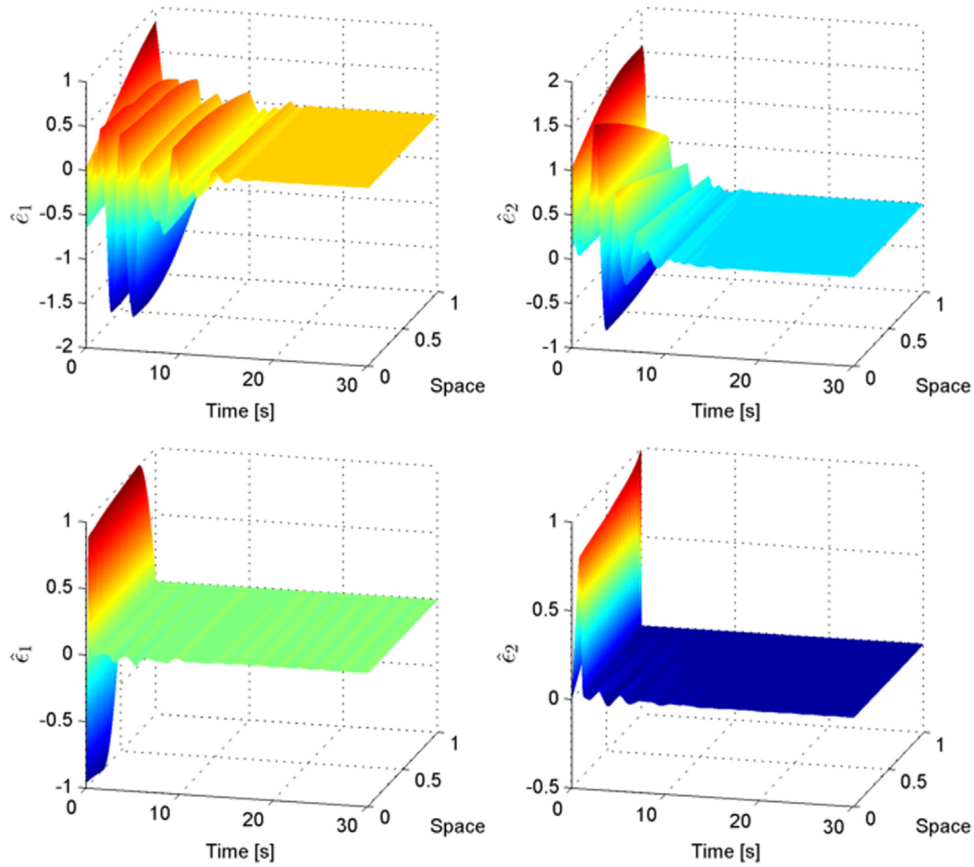

Fig. 4. Prediction errors, observer of Theorem 8.

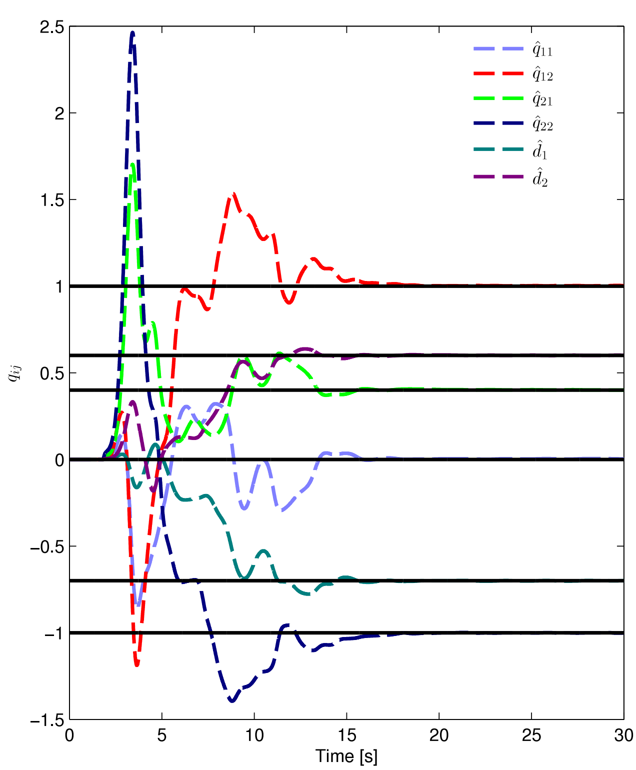

Fig. 5. Estimated and actual boundary parameters, observer of Theorem 8.

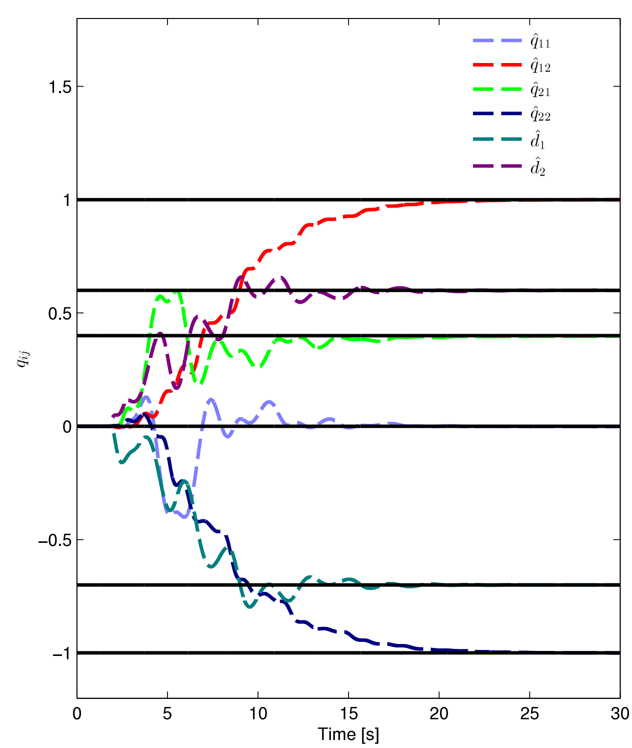

Fig. 6. Estimated and actual boundary parameters, observer of Theorem 12.

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Appendix A. Observer kernels equations

A.1. Kernel equations

The kernel equations are

$$\Lambda^+ M_x(x,\xi) + M_\xi(x,\xi)\Lambda^+ = \Sigma^{++}M(x,\xi) + \Sigma^{+-}N(x,\xi) - M(x,\xi)\bar{\Sigma}^{++} \quad \text{(A.1)}$$

$$-\Lambda^- N_x(x,\xi) + N_\xi(x,\xi)\Lambda^+ = \Sigma^{-+}M(x,\xi) + \Sigma^{--}N(x,\xi) - N(x,\xi)\bar{\Sigma}^{++} \quad \text{(A.2)}$$

with boundary conditions

$$\Lambda^+ M(x,x) - M(x,x)\Lambda^+ + \Sigma^{++} = \bar{\Sigma}^{++} \quad \text{(A.3)}$$

$$\Lambda^- N(x,x) + N(x,x)\Lambda^+ - \Sigma^{-+} = 0 \quad \text{(A.4)}$$

with

$$M_{ij}(0,\xi) = 0, \quad \text{for } 1 \le i \le j \le n \quad \text{(A.5)}$$

and

$$M_{ij}(x,1) = \frac{\sigma_{ij}^{++}}{\lambda_j - \lambda_i}, \quad \text{for } 1 \le j < i \le n. \quad \text{(A.6)}$$

The well-posedness of these equations is addressed in the next section.

A.2. Well-posedness

We will prove the existence of a solution of the kernel equations (A.1)–(A.6) by transforming them into the form treated in Hu et al. (2016). Consider the transformation

$$\bar{M}_{ij}(\chi,y) = M_{ij}(y,\chi) \Leftrightarrow M_{ij}(x,\xi) = \bar{M}_{ij}(\xi,x) \quad \text{(A.7)}$$

$$\bar{N}_{ij}(\chi,y) = N_{ij}(y,x) \Leftrightarrow N_{ij}(x,\xi) = \bar{N}_{ij}(\xi,x). \quad \text{(A.8)}$$

Then Eqs. (A.1)–(A.6), when written out in their components, become, for $1 \le i,j \le n$

$$\lambda_j \partial_\chi \bar{M}_{ij}(\chi,y) + \lambda_i \partial_y \bar{M}_{ij}(\chi,y) = \sum_{k=1}^{n} \sigma_{ik}^{++}\bar{M}_{kj}(\chi,y) + \sum_{k=1}^{m} \sigma_{ik}^{+-}\bar{N}_{kj}(\chi,y) - \sigma_{jj}^{++}\bar{M}_{ij}(\chi,y) \quad \text{(A.9)}$$

and for $1 \le i \le m$, $1 \le j \le n$

$$\lambda_j \partial_\chi \bar{N}_{ij}(\chi,y) - \mu_i \partial_y \bar{N}_{ij}(\chi,y) = \sum_{k=1}^{n} \sigma_{ik}^{-+}\bar{M}_{kj}(\chi,y) + \sum_{k=1}^{m} \sigma_{ik}^{--}\bar{N}_{kj}(\chi,y) - \sigma_{jj}^{++}\bar{N}_{ij}(\chi,y), \quad \text{(A.10)}$$

with boundary conditions, for $1 \le i,j \le n, i \ne j$

$$\bar{M}_{ij}(\chi,\chi) = \frac{\sigma_{ij}^{++}}{\lambda_j - \lambda_i} \quad \text{(A.11)}$$

and $1 \le i \le m$ $1 \le j \le n$

$$\bar{N}_{ij}(\chi,\chi) = \frac{\sigma_{ij}^{-+}}{\mu_i + \lambda_j} \quad \text{(A.12)}$$

while for $1 \le i \le j \le n$

$$\bar{M}_{ij}(\chi,0) = 0 \quad \text{(A.13)}$$

with, for $1 \le j < i \le n$

$$\bar{M}_{ij}(1,y) = \frac{\sigma_{ij}^{++}}{\lambda_j - \lambda_i}. \quad \text{(A.14)}$$

The kernel equations (A.9)–(A.14) have the same form as the $K - L$ kernels in Hu et al. (2016), for which well-posedness was proved.

Appendix B. Additional lemma

Lemma 13. *Consider the matrices*

$$A \in \mathbb{R}^{m\times n}, \qquad B \in \mathbb{R}^{m\times m}, \qquad Q \in \mathbb{R}^{n\times m} \quad \text{(B.1)}$$

and assume the matrix $AQ + B$ is invertible. Then, the matrix

$$F = I_{n\times n} - Q(AQ+B)^{-1}A \quad \text{(B.2)}$$

is invertible if and only if B is invertible.

Proof (*Proof originally from daw, 2015*)**.** Assume B is invertible. Then

$$\begin{aligned} AF &= A - AQ\,(AQ+B)^{-1}A \\ &= A - (AQ+B-B)\,(AQ+B)^{-1}A \\ &= B\,(AQ+B)^{-1}A. \end{aligned} \quad \text{(B.3)}$$

Let x be such that $Fx = 0$. It follows that $AFx = 0$, and thus, from (B.3)

$$B\,(AQ+B)^{-1}Ax = 0. \quad \text{(B.4)}$$

Since $AQ + B$ and B are both nonsingular, $Ax = 0$ must hold. Then

$$Fx = (I_{n\times n} - Q(AQ+B)^{-1}A)x = x \quad \text{(B.5)}$$

and hence $x = 0$, which means that F is invertible.

Next, assume F is invertible. Then

$$\begin{aligned} FQ &= Q - Q(AQ+B)^{-1}AQ \\ &= Q - Q(AQ+B)^{-1}(AQ+B-B) \\ &= Q(AQ+B)^{-1}B. \end{aligned} \quad \text{(B.6)}$$

Let x be such that $Bx = 0$. Then it follows from (B.6) that $FQx = 0$, and since F is invertible, that is $Qx = 0$. Knowing that $Qx = 0$ and $Bx = 0$, we deduce

$$(AQ+B)x = 0. \quad \text{(B.7)}$$

Since $AQ + B$ is invertible, $x = 0$, and hence B is invertible. □

References

Aamo, Ole Morten (2013). Disturbance rejection in 2 × 2 linear hyperbolic systems. *IEEE Transactions on Automatic Control*, *58*(5), 1095–1106.

Aamo, Ole Morten (2016). Leak detection, size estimation and localization in pipe flows. *IEEE Transactions on Automatic Control*, *61*(1), 246–251.

Aamo, Ole Morten, Smyshlyaev, Andrey, Krstić, Miroslav, & Foss, Bjarne Anton (2004). Stabilization of a Ginzburg-Landau model of vortex shedding by output feedback boundary control. In *CDC. 43rd IEEE conference on decision and control, Vol. 3* (pp. 2409–2416).

Amin, Saurabh, Hante, Falk M., & Bayen, Alexandre M. (2008). On stability of switched linear hyperbolic conservation laws with reflecting boundaries. In *Hybrid systems: computation and control* (pp. 602–605). Springer-Verlag.

Anfinsen, Henrik, & Aamo, Ole Morten (2015). Disturbance rejection in the interior domain of linear 2 x 2 hyperbolic systems. *IEEE Transactions on Automatic Control*, *60*(1), 186–191.

Anfinsen, Henrik, Diagne, Mamadou, Aamo, Ole Morten, & Krstić, Miroslav (2016). An adaptive observer design for n + 1 coupled linear hyperbolic PDEs based on swapping. *IEEE Transactions on Automatic Control*, *61*(12), 3979–3990.

Anfinsen, Henrik, Di Meglio, Florent, & Aamo, Ole Morten (2016). Estimating the left boundary condition of coupled 1-D linear hyperbolic PDEs from right boundary sensing. In *15th European control conference* (pp. 2179–2184).

Bernard, Pauline, & Krstić, Miroslav (2014). Adaptive output-feedback stabilization of non-local hyperbolic PDEs. *Automatica*, *50*, 2692–2699.

Bin, Michelangelo, & Di Meglio, Florent (2015). Boundary estimation of parameters for linear hyperbolic PDEs. *IEEE Transactions on Automatic Control* (in press).

Bosković, Dejan M., Krstić, Miroslav, & Liu, Weijiu (2001). Boundary control of an unstable heat equation via measurement of domain-averaged temperature. *IEEE Transactions on Automatic Control*, *46*(12), 2022–2028.

Coron, Jean-Michel, Novel, Brigitte d'Andrea, & Bastin, Georges (2007). A strict lyapunov function for boundary control of hyperbolic systems of conservation laws. *IEEE Transactions on Automatic Control*, *52*(1), 2–11.

Curró, C., Fusco, D., & Manganaro, N. (2011). A reduction procedure for generalized Riemann problems with application to nonlinear transmission lines. *Journal of Physics A: Mathematical and Theoretical*, *44*(33), 335205.

daw (2015). (http://math.stackexchange.com/users/136544/daw). Nonsingularity of matrix combination. Mathematics Stack Exchange. http://math.stackexchange.com/q/1536726. Downloaded November 20 2015.

Diagne, Ababacar, Diagne, Mamadou, Tang, Shuxia, & Krstić, Miroslav (2017). Backstepping stabilization of the linearized Saint-Venant-Exner model. *Automatica*, *76*, 345–354.

Gou, Bao-Zhu, & Jin, Feng-Fei (2015). Output feedback stabilization for one-dimensional wave equation subject to boundary disturbance. *IEEE Transactions on Automatic Control*, *60*(3), 824–830.

Greenberg, J. M., & Tsien, Li Ta (1984). The effect of boundary damping for the quasilinear wave equation. *Journal of Differential Equations*, *52*(1), 66–75.

Hu, Long, Di Meglio, Florent, Vazquez, Rafael, & Krstić, Miroslav (2016). Control of homodirectional and general heterodirectional linear coupled hyperbolic PDEs. *IEEE Transactions on Automatic Control*, *61*(11), 3301–3314.

Ioannou, Petros, & Sun, Jing (1995). *Robust adaptive control*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc..

Krstić, Miroslav, Kanellakopoulos, Ioannis, & Kokotović, Petar V. (1995). *Nonlinear and adaptive control design*. Wiley.

Krstić, Miroslav, & Smyshlyaev, Andrey (2008). Backstepping boundary control for first-order hyperbolic PDEs and application to systems with actuator and sensor delays. *Systems & Control Letters*, *57*(9), 750–758.

Landet, Ingar Skyberg, Pavlov, Alexey, & Aamo, Ole Morten (2013). Modeling and control of heave-induced pressure fluctuations in managed pressure drilling. *IEEE Transactions on Control Systems and Technology*, *21*(4), 1340–1351.

Litrico, Xavier, & Fromion, Vincent (2006). Boundary control of hyperbolic conservation laws with a frequency domain approach. In *Proceedings of the 45th IEEE conference on decision & control, Manchester Grand Hyatt Hotel*.

Liu, Weijiu (2003). Boundary feedback stabilization of an unstable heat equation. *SIAM Journal on Control and Optimization*, *42*, 1033–1043.

Di Meglio, Florent (2011). *Dynamics and control of slugging in oil production*. (Ph.D. thesis) MINES ParisTech.

Di Meglio, Florent, Bresch-Pietri, Delphine, & Aarsnes, Ulf Jakob F. (2014). An adaptive observer for hyperbolic systems with application to under balanced drilling. In *IFAC world congress 2014* (pp. 11391–11397).

Di Meglio, Florent, Vazquez, Rafael, & Krstić, Miroslav (2013). Stabilization of a system of n + 1 coupled first-order hyperbolic linear PDEs with a single boundary input. *IEEE Transactions on Automatic Control*, *58*(12), 3097–3111.

Smyshlyaev, Andrey, Cerpa, Eduardo, & Krstić, Miroslav (2010). Boundary stabilization of a 1-d wave equation with in-domain antidamping. *SIAM Journal on Control and Optimization*, *48*(6), 4014–4031.

Smyshlyaev, Andrey, & Krstić, Miroslav (2007). Adaptive boundary control for unstable parabolic PDEs - Part III: Output feedback examples with swapping identifiers. *Automatica*, *43*, 1557–1564.

Smyshlyaev, Andrey, & Krstić, Miroslav (2010). *Adaptive control of parabolic PDEs*. Princeton University Press.

Tang, Shuxia, & Krstić, Miroslav (2014). Sliding mode control to the stabilization of a linear 2 x 2 hyperbolic system with boundary input disturbance. In *American control conference 2014*.

Vazquez, Rafael, & Krstić, Miroslav (2008a). Control of 1-D parabolic PDEs with Volterra nonlinearities, Part I: Design. *Automatica*, *44*, 2778–2790.

Vazquez, Rafael, & Krstić, Miroslav (2008b). Control of 1-D parabolic PDEs with Volterra nonlinearities, Part II: Analysis. *Automatica*, *44*, 2791–2803.

Vazquez, Rafael, Krstić, Miroslav, & Coron, Jean-Michel (2011). Backstepping boundary stabilization and state estimation of a 2×2 linear hyperbolic system. In *2011 50th IEEE conference on decision and control and European control conference, CDC-ECC* (pp. 4937–4942).

Xu, Zaihua, & Liu, Yungang (2016). Adaptive boundary stabilization for first-order hyperbolic PDEs with unknown spatially varying parameter. *International Journal of Robust and Nonlinear Control*, *26*(3), 613–628.

Xu, Cheng-Zhong, & Sallet, Gauthier (2010). Exponential stability and transfer functions of processes governed by symmetric hyperbolic systems. *ESAIM: Control, Optimisation and Calculus of Variations*, *7*, 421–442.

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