



Event-triggered boundary control of a continuum model of highly re-entrant manufacturing systems[☆]

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ARTICLE INFO

Article history:

Received 4 April 2020

Received in revised form 14 May 2021

Accepted 16 July 2021

Available online 21 September 2021

Keywords:

Nonlocal PDEs

Event-triggered control

Lyapunov design

Manufacturing systems

Sampled-data control

ABSTRACT

We design an event-triggered boundary controller for a continuum model of highly re-entrant manufacturing systems for which the influx rate of products is the controlled quantity. Mathematically, the governing equation of the process is a nonlinear hyperbolic partial differential equation with a nonlinearity arising from the nonlocal propagation speed and the positivity state constraint. The nonlinearity makes highly non-trivial the proof of well-posedness of the closed-loop system under ETC, and the design of the triggering mechanism requires novelty because standard triggering schemes proposed in the literature of linear PDEs do not work. The designed controller can potentially operate in networked control systems subject to limited information sharing resources. A Lyapunov argument is utilized to derive the boundary controller together with a feasible event generator that avoids the occurrence of Zeno behavior for the closed-loop system. The global stability estimate is established using the logarithmic norm of the state due to the system's nonlinearity and positivity of the density. Furthermore, robustness of the proposed controller with respect to the sampling schedule and sampled-data stabilization results are established. Consistent simulation results that support the proposed theoretical statements are provided.

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1. Introduction

The comprehensive development of event-triggered control in the past few years has been motivated by the crucial need to preserve limited computational and communication resources during the execution of feedback control tasks in constrained Networked Control Systems (NCS). The key idea of event-triggered control consists of the starting of “only necessary” control action when events generated by the real-time systems’ response occur, reducing considerably the number of execution of control tasks while preserving satisfactory closed-loop system performance. In comparison to the well-known periodical sampled-data approach, event-triggered enables noncyclic updates of control signals and thereby offers more flexibility to manage constraints arising from networks and systems interactions. More precisely, periodic sampling of control inputs is often undesirable due to bandwidth constraints in the communication and task scheduling

limitations. Generally speaking, event-based control designs require two challenging steps that can be achieved simultaneously or independently:

- The construction of a feasible successive execution time sequence (with positive minimum dwell-time), which is related to the event generator that determines the time instants at which the control input is updated.
- The design of the feedback control signal that ensures closed-loop performance specifications.

Besides the study of digital computer design for closed loop systems (Hsu & Sastry, 1987; Monaco & Normand-Cyrot, 1985), the event-based PID control design proposed in Åarzén (1999) and the event-based sampling for first-order stochastic systems (Åström & Bernhardsson, 1999) serve as pioneering contributions in the field. These results consider scheduling algorithms that only recondition feedback control signals according to an error with respect to a given state norm. Later on, motivated by the stabilization of constrained networked control systems (Hespanha, Naghshtabrizi, & Xu, 2007), major results dedicated to both linear (Heemels, Donkers, & Teel, 2012; Heemels, Johansson, & Tabuada, 2012; Nowzari, Garcia, & Cortés, 2019; Peng & Han, 2013; Yook, Tilbury, & Soparkar, 2002) and non-linear (Abdelrahim, Postoyan, Daafouz, & Nešić, 2016; Nowzari et al., 2019; Postoyan, Tabuada, Nešić, & Anta, 2015; Seuret &

[☆] The material in this paper was partially presented at the 2021 American Control Conference, May 25–28, 2021, New Orleans, LA, USA. This paper was recommended for publication in revised form by Associate Editor Aneel Tanwani under the direction of Editor Sophie Tarbouriech.

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Prieur, 2011; Tabuada, 2007; Tallapragada & Chopra, 2013) finite-dimensional systems have been established introducing Lyapunov-based triggering conditions to ensure stability at desired decay rate. As well, advanced control designs involving robustness analysis (Borgers & Heemels, 2014; Hetel et al., 2017), stabilization of multi-agent consensus problems and adaptive control (Xing, Wen, Liu, Su, & Cai, 2017) can be found in the abundant literature. From a practical point of view, real implementation of event-based schemes has been achieved to control wireless throttling valves (Blevins, Nixon, & Wojsznis, 2015), the angular position of a gyroscope (Boisseau, Dur, Martinez-Molina, Raharijaona, & Marchand, 2015), the formation of a group of VTOL-UAVs (Guerrero-Castellanos et al., 2017) and a greenhouse temperature (Pawlowski, Guzmàn, Berenguel, & Dormido, 2015), to name a few.

Concurrently, substantial efforts have been taken to develop event-based control of infinite-dimensional systems. In this case, the deduction of the triggering condition follows a Lyapunov criterion and is analogous to that of finite-dimensional systems. Early results are developed upon reduced-order models that describe the dominant dynamics of reaction-diffusion systems. In this case, the resulting linear finite-dimensional systems are exploited to match the control objectives (Xue & El-Farray, 2017; Yao & El-Farray, 2012). However, it is well known that the order of approximation is not trivially determined a priori using a modal expansion of PDEs (Partial Differential Equations). Alongside, both event-triggered and sampled-data control have been successfully developed for reaction-advection-diffusion PDE (Espitia, Karafyllis, & Krstic, 2021; Karafyllis & Krstic, 2018; Selivanov & Fridman, 2015, 2016; Wang, 2019; Wang & Wang, 2019) and ODE-PDE cascading systems (Ahmed-Ali, Karafyllis, Giri, Krstic, & Lamnabhi-Lagarigue, 2017) without model reduction. We emphasize that Espitia et al. (2021), which employ a small gain design, is one of the first attempt of event-triggered boundary control of 1D parabolic PDEs. Furthermore, Rathnayake, Diagne, Espitia, and Karafyllis (2020) proposed the infinite-dimensional event-based output feedback boundary control design for parabolic PDEs using boundary measurements. For hyperbolic PDEs substantial developments can be found in Baudouin, Marx, and Tarbouriech (2019), Davo, Bresch-Pietri, Prieur, and Di Meglio (2018), Espitia (2020), Espitia, Girard, March, and Prieur (2016, 2018), Espitia, Tanwani, and Tarbouriech (2017) and Karafyllis and Krstic (2017a). It is worth to mention that local stability result has been recently achieved applying sampled-data control to a nonlinear PDE governed by 1-D Kuramoto-Sivashinsky equation (Kang & Fridman, 2018).

Furthermore, as emphasized in the premier contribution (Árzn, 1999), for factory lines, the event-based nature of the sampling can be related to the process's intrinsic production rate. From a modeling point of view, discrete-event (Hu & Zhou, 2014; Zhou & DiCesare, 2012), Effective Processing Time (EPT) (Jacobs, Etman, Van Campen, & Rooda, 2003) and clearing functions (Graves, 1986) representations, which requires a deep knowledge of various process specifications or only accounts on arrival and departure events of the parts to a workstation when Work in Progress (WIP) is not strongly varying, has been proven inefficient for operation planning and control. Motivated by the study of the transient behavior of manufacturing systems with high fluctuations of the WIP, non-local transport PDE models (Armbruster, Göttlich, & Herty, 2011; Armbruster, Marthaler, & Ringhofer, 2003; Lefeber, Van Den Berg, & Rooda, 2004) have emerged during the past few years. These continuum models describe the time evolution of the flow of manufactured products using the spatial distribution of product density as a key variable. Several contributions considering the control of boundary influx of parts with PI controller (Xu, Ni,

Yuan, & Dubljevic, 2018), Lyapunov-based design (Coron & Wang, 2012, 2013; Shang & Wang, 2011), small gain design (Karafyllis & Krstic, 2020), predictor-feedback design (Diagne, Bekiaris-Liberis, & Krstic, 2017) or optimal (D'Apice, Kogut, & Manzo, 2016; La Marca, Armbruster, Herty, & Ringhofer, 2010) control techniques of these non-local PDEs have been recently developed by researchers.

With the unceasing growth of intelligent production lines that integrate sensors, actuators, and controllers in a wireless communication environment via internet of things (IoT), we design an event-triggered boundary controller for a continuum model of highly re-entrant manufacturing systems for which the influx rate of product is the controlled quantity. Mathematically, the governing equation of the process is a nonlinear hyperbolic partial differential equation with a nonlinearity arising from the non-local propagation speed and the positivity state constraint. The nonlinearity makes highly non-trivial the proof of well-posedness of the closed-loop system under ETC, and the design of the triggering mechanism requires novelty because standard triggering schemes proposed in the literature of linear PDEs do not work. The designed controller can potentially operate in networked control systems subject to limited information sharing resources. A Lyapunov argument is utilized to derive the boundary controller together with a feasible event generator that avoids the occurrence of Zeno behavior for the closed-loop system. The global stability estimate is established using the logarithmic norm of the state due to the system's nonlinearity and positivity of the density. Furthermore, robustness of the proposed controller with respect to the sampling schedule and sampled-data stabilization results (Theorem 4) are established. Consistent simulation results that support the proposed theoretical statements are provided.

The paper is organized as follows: Section 2 describes the modeling of highly re-entrant manufacturing systems as nonlocal PDE with an influx boundary condition. The existence and uniqueness of solutions for a piecewise continuous control input signal are stated in Section 3. The construction of robust event-triggered and sampled-data stabilizing boundary controllers is discussed in Section 4. Simulation results that demonstrate the feasibility of both event-triggered and sampled-data controllers are discussed in Section 5. Finally, the paper ends with concluding remarks and future research directions in Section 6.

Notation: Throughout the paper, we adopt the following notation.

- $\mathbb{R}_+ := [0, +\infty)$. Let $u : \mathbb{R}_+ \times [0, 1] \rightarrow \mathbb{R}$ be given. We use the notation $u[t]$ to denote the profile at certain $t \geq 0$, i.e., $(u[t])(x) = u(t, x)$ for all $x \in [0, 1]$. For bounded functions $f : I \rightarrow \mathbb{R}$, where $I \subseteq \mathbb{R}$ is an interval, we set $\|f\|_\infty = \sup_{x \in I} (|f(x)|) < +\infty$. We use the notation $f'(x)$ for the derivative at $x \in [0, 1]$ of a differentiable function $f : [0, 1] \rightarrow \mathbb{R}$.
- Let $S \subseteq \mathbb{R}^n$ be an open set and let $A \subseteq \mathbb{R}^n$ be a set that satisfies $S \subseteq A \subseteq \text{cl}(S)$. By $C^0(A; \Omega)$, we denote the class of continuous functions on A , which take values in $\Omega \subseteq \mathbb{R}^m$. By $C^k(A; \Omega)$, where $k \geq 1$ is an integer, we denote the class of functions on $A \subseteq \mathbb{R}^n$, which takes values in $\Omega \subseteq \mathbb{R}^m$ and has continuous derivatives of order k . In other words, the functions of class $C^k(A; \Omega)$ are the functions which have continuous derivatives of order k in $S = \text{int}(A)$ that can be continued continuously to all points in $\partial S \cap A$. When $\Omega = \mathbb{R}$ then we write $C^0(A)$ or $C^k(A)$.
- A left-continuous function $f : [0, 1] \rightarrow \mathbb{R}$ (i.e. a function with $\lim_{y \rightarrow x^-} (f(y)) = f(x)$ for all $x \in (0, 1]$) is called piecewise C^1 on $[0, 1]$ and we write $f \in PC^1([0, 1])$, if the following properties hold: (i) for every $x \in [0, 1)$ the limits $\lim_{y \rightarrow x^+} (f(y))$, $\lim_{h \rightarrow 0^+, y \rightarrow x^+} (h^{-1} (f(y+h) - f(y)))$ exist and are finite, (ii)

for every $x \in (0, 1]$ the limit $\lim_{h \rightarrow 0^-} (h^{-1}(f(x+h) - f(x)))$ exists and is finite, (iii) there exists a set $J \subset (0, 1)$ of finite cardinality, where $f'(x) = \lim_{h \rightarrow 0^-} (h^{-1}(f(x+h) - f(x))) = \lim_{h \rightarrow 0^+} (h^{-1}(f(x+h) - f(x)))$ holds for $x \in (0, 1) \setminus J$, and (iv) the mapping $((0, 1) \setminus J) \ni x \rightarrow f'(x) \in \mathbb{R}$ is continuous. Notice that we require a piecewise C^1 function to be left-continuous but not continuous.

2. Continuum model of highly re-entrant manufacturing systems

Manufacturing systems with a high volume and a large number of consecutive production steps (which typically number in the many hundreds) are often modeled by non-local PDEs (Armbruster et al., 2011, 2003; Lefeber et al., 2004). The general one-dimensional continuity equation expressing mass conservation along the production stages is considered. Defining the flow of unit parts per unit time as $(F(\rho[t]))(x)$ where F is the flux at the production stage x and time t depending of the density of product $\rho(t, x)$, namely, the work in progress, the following equation of conservation can be written between two production stages x_1 and x_2

$$\frac{\partial}{\partial t} \int_{x_1}^{x_2} \rho(t, x) dx = (F(\rho[t]))(x_1) - (F(\rho[t]))(x_2), \tag{1}$$

and equivalently in differential form as

$$\frac{\partial \rho(t, x)}{\partial t} + \frac{\partial (F(\rho[t]))(x)}{\partial x} = 0. \tag{2}$$

The density function $\rho(t, x)$ defined at time $t \geq 0$ and stage $x \in [0, 1]$, is required to be positive (i.e., $\rho(t, x) > 0$ for $(t, x) \in \mathbb{R}_+ \times [0, 1]$). Moreover, it has a spatially uniform and positive equilibrium profile

$$\rho(x) \equiv \rho_s, \quad \rho_s > 0.$$

Controlling of the governing equation (2) consists of determining the influx

$$(F(\rho[t]))(0) = u(t) \tag{3}$$

that results in the desired outflux

$$(F(\rho[t]))(1) = y(t). \tag{4}$$

A challenging aspect of the model (2) is the characterization of flux F as a function of the work in progress ρ . Following Armbruster et al. (2011, 2003) and Lefeber et al. (2004), F can be defined as a function of the length of the queue or the total work load $W(t)$, that is,

$$(F(\rho[t]))(x) = \lambda(W(t)) \rho(t, x),$$

leading to the following equation

$$\frac{\partial \rho}{\partial t}(t, x) + \lambda(W(t)) \frac{\partial \rho}{\partial x}(t, x) = 0, \tag{5}$$

where

$$W(t) = \int_0^1 \rho(t, x) dx. \tag{6}$$

Here, $\lambda \in C^1(\mathbb{R}_+; (0, +\infty))$ is a non-increasing function that determines the production speed. Hence, the process influx rate at the boundary defined in (3) becomes

$$\rho(t, 0) \lambda(W(t)) = u(t), \tag{7}$$

where $u(t) \in (0, +\infty)$ is the control input.

In what follows, we intend to apply event-triggered boundary control through $u(t) \in (0, +\infty)$ to the manufacturing plant described by the (5)–(7), which will achieve global stabilization

of the spatially uniform equilibrium profile $\rho(x) \equiv \rho_s$. Therefore, the control $u(t) \in (0, +\infty)$ will have the form

$$u(t) = u_i, \text{ for } t \in [t_i, t_{i+1}) \tag{8}$$

where $u_i > 0$ are the input values that will be determined by the controller and the times $\{t_i : i = 0, 1, 2, \dots\}$ will be the times of the events that will be determined by the event-trigger, which will constitute an increasing sequence with $t_0 = 0$.

3. Notion of solution

First we describe precisely the notion of the solution of the closed-loop system.

3.1. Existence and uniqueness of solution under constant boundary input signal

The following result plays an instrumental role in the construction of solutions for the closed-loop system (5), (6), (7), (8).

Theorem 1. Consider the initial-boundary value problem (5), (6), (7) with initial condition

$$\rho(0, x) = \rho_0(x), \text{ for } x \in (0, 1] \tag{9}$$

where $\lambda \in C^1(\mathbb{R}_+; (0, +\infty))$ is a non-increasing function, $\rho_0 \in PC^1([0, 1])$, $u(t) \equiv u > 0$ and $\inf_{x \in (0, 1)} (\rho_0(x)) > 0$. Suppose that there exists a constant $K > 0$ such that $|\lambda'(s)| \leq K$ for all $s \geq 0$. Then there exist $t_{\max} \in (0, +\infty)$ and unique functions $W \in C^0([0, t_{\max}); (0, +\infty))$, $\rho : [0, t_{\max}) \times [0, 1] \rightarrow (0, +\infty)$ with $\rho[t] \in PC^1([0, 1])$ and $\inf_{x \in (0, 1)} (\rho(t, x)) > 0$ for all $t \in [0, t_{\max})$, which constitute a solution of the initial-boundary value problem (5), (6), (7), (9) in the following sense:

- The function $\rho : [0, t_{\max}) \times [0, 1] \rightarrow (0, +\infty)$ is of class $C^1([0, t_{\max}) \times [0, 1] \setminus \Omega)$, where

$$\Omega = \Omega_1 \cup \Omega_2 \tag{10}$$

with

$$\Omega_1 = \bigcup_{i=0, \dots, N} \{(t, r_i(t)) : t \in [0, t_{\max}), r_i(t) \leq 1\}, \tag{11}$$

$$\Omega_2 = \bigcup_{i=0, \dots, N} \left\{ (t, r_i(t) - 1) : t \in [0, t_{\max}), \right. \\ \left. 1 \leq r_i(t) \leq 2 \right\}, \tag{12}$$

and

$$r_i(t) = \xi_i + \int_0^t \lambda(W(s)) ds, \quad (i = 0, \dots, N), \tag{13}$$

$\xi_0 = 0$ and $\xi_i \in [0, 1]$ ($i = 1, \dots, N$) are the points (in increasing order) for which $\rho_0 \in C^1([0, 1] \setminus \{\xi_0, \dots, \xi_N\})$.

- Eq. (5) holds for all $(t, x) \in [0, t_{\max}) \times [0, 1] \setminus \Omega$ and Eqs. (6), (7), (9) hold for $t \in [0, t_{\max})$ with $u(t) \equiv u > 0$.

- Finally, if $t_{\max} < +\infty$ then

$$\limsup_{t \rightarrow t_{\max}^-} (\|\rho[t]\|_\infty) = +\infty. \tag{14}$$

Remark 1. Theorem 1 whose detailed proof is given in Section Appendix, states the existence of solution of the initial-boundary value problem (5), (6), (7), (9) with a positive piecewise continuous initial data and a positive constant input $u(t) = u$. Note that Theorem 1 does not exclude the possible existence of a finite escape time, i.e., $t_{\max} < +\infty$. However, it shows that if a finite escape time exists, then the solution must “blow up”.

3.2. Existence and uniqueness of solution under an event-triggered boundary control law

Given $\rho_0 \in PC^1([0, 1])$ with $\inf_{x \in (0,1)}(\rho_0(x)) > 0$ and using [Theorem 1](#), we are in a position to construct a unique solution for the closed-loop system (5), (6), (7), (8), (9) by means of the following algorithm for each integer $i \geq 0$:

- (1) Given $t_i \geq 0$ and $\rho[t_i] \in PC^1([0, 1])$ with $\inf_{x \in (0,1)}(\rho(t_i, x)) > 0$, determine $u_i > 0$ by using an appropriate feedback law.
- (2) Construct the solution of the initial-boundary value problem

$$\frac{\partial y}{\partial t}(t, x) + \lambda(V(t)) \frac{\partial y}{\partial x}(t, x) = 0,$$

$$V(t) = \int_0^1 y(t, x) dx,$$

$$y(t, 0)\lambda(V(t)) = u_i,$$

with initial condition $y(0, x) = \rho(t_i, x)$, for $x \in (0, 1]$. [Theorem 1](#) guarantees that such a solution exists as long as y remains bounded in the interval $[0, t_{i+1} - t_i]$.

- (3) Using the event-trigger and assuming that $\rho[t_i + s] = y[s]$ for $s \geq 0$, determine the time of the next event $t_{i+1} > t_i$.
- (4) Set $\rho[t_i + s] = y[s]$ for $0 \leq s \leq t_{i+1} - t_i$ and repeat.

The above algorithm guarantees that the solution satisfies $\rho[t] \in PC^1([0, 1])$ and $\inf_{x \in (0,1)}(\rho(t, x)) > 0$ for all $t \in [0, t_{\max}]$, where $t_{\max} \in (0, +\infty]$ is the maximal existence time of the solution. Moreover, Eqs. (6), (7), (8), (9) hold, while the PDE (5) holds for $(t, x) \in [0, t_{\max}] \times [0, 1]$ almost everywhere. Finally, if $t_{\max} < \lim_{i \rightarrow +\infty}(t_i)$ then $\limsup_{t \rightarrow t_{\max}^-}(\|\rho[t]\|_{\infty}) = +\infty$, which implies that the solution exists as long as it is bounded.

4. Event-triggered and sampled-data control design

4.1. Design of the event-triggered boundary control law

We design an event-triggering control law by defining a static triggering condition and a control signal based on a Lyapunov argument.

Let $\sigma > 0$ be a parameter of the controller and consider the event-triggered controller given by the following formulas for all integers $i \geq 0$:

$$t_0 = 0 \tag{15}$$

$$\mu_i = \inf \left\{ \tau > t_i : \Psi(\tau, 0) > \exp \left(-\sigma \int_{t_i}^{\tau} \lambda(W(s)) ds \right) \sup_{0 < x \leq 1} (\Psi(t_i, x) \exp(-\sigma x)) \right\} \tag{16}$$

$$t_{i+1} = \min \left(t_i + \frac{1}{\lambda(0)}, \mu_i \right), \tag{17}$$

$$u_i = \rho_s \lambda(W(t_i)), \tag{18}$$

where

$$\Psi(t, x) = \left| \ln \left(\frac{\rho(t, x)}{\rho_s} \right) \right|. \tag{19}$$

It is possible to show that the event-triggered controller (8), (15), (16), (17), (18) guarantees global stabilization of system (5), (6), (7). Moreover, no Zeno behavior can occur for the closed-loop system. The structure of the closed-loop system consisting of the plant, the event generator, and the control input is depicted in [Fig. 1](#).

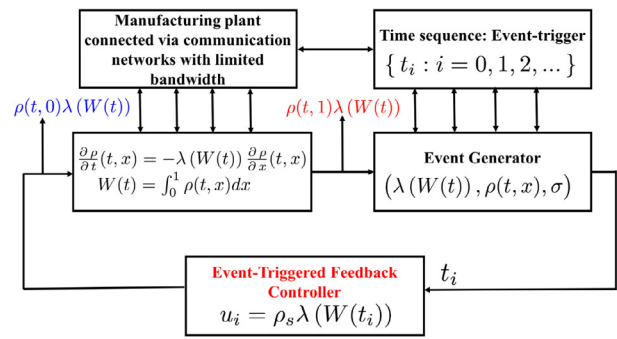


Fig. 1. Event-trigger closed-loop system.

Remark 2. The stability estimate is established in a special state norm. Due to the positivity of the state, the logarithmic norm of the state ρ appears, i.e., we have $\Psi(t, x)$ given in (19) instead of the usual $|\rho(t, x) - \rho_s|$ that appears in many stability estimates for linear PDEs. The logarithmic norm is a manifestation of the nonlinearity of system (5), (6), (7) and the fact that the state space is not a linear space but rather a positive cone: the state space for system (5), (6), (7) is the set $X := \{\rho \in PC^1([0, 1]) : \inf_{x \in (0,1)}(\rho(x)) > 0\}$. The use of the logarithmic norm of the state is common in systems with positivity constraints (see [Karafyllis & Krstic, 2017b, 2020](#)).

Next, we state our main result in the following theorem.

Theorem 2. Suppose that there exists a constant $K > 0$ such that $|\lambda'(s)| \leq K$ for all $s \geq 0$. Let $\sigma > 0$ be a given parameter and consider the closed-loop system (5), (6), (7), (8), (15), (16), (17), (18). Then there exists a non-increasing function $T : \mathbb{R}_+ \rightarrow (0, +\infty)$ such that for every $\rho_0 \in PC^1([0, 1])$ with $\inf_{x \in (0,1)}(\rho_0(x)) > 0$, the following estimates hold for the solution $\rho : \mathbb{R}_+ \times [0, 1] \rightarrow (0, +\infty)$ with $\rho[t] \in PC^1([0, 1])$ and $\inf_{x \in (0,1)}(\rho(t, x)) > 0$ for all $t \geq 0$ of the initial-boundary value problem (5), (6), (7), (8), (9), (15), (16), (17), (18):

$$\sup_{0 \leq x \leq 1} \Psi(t, x) \leq \exp(-\sigma(c(\rho_0)t - 1)) \sup_{0 < x \leq 1} \Psi_0(x), \tag{20}$$

for all $t \geq 0$,

$$t_{i+1} \geq t_i + T \left(\sup_{0 < x \leq 1} \Psi_0(x) \right), \text{ for all } t \geq 0, \tag{21}$$

where

$$c(\rho_0) := \lambda \left(\rho_s \exp \left(\exp(\sigma) \sup_{0 < x \leq 1} \Psi_0(x) \right) \right) \tag{22}$$

and $\Psi(t, x)$ is given in (19).

Proof of Theorem 2. Let $\rho_0 \in PC^1([0, 1])$ with $\inf_{x \in (0,1)}(\rho_0(x)) > 0$ be given (arbitrary). By virtue of [Theorem 1](#), there exists a unique solution $\rho : [0, t_{\max}] \times [0, 1] \rightarrow (0, +\infty)$ with $\rho[t] \in PC^1([0, 1])$ and $\inf_{x \in (0,1)}(\rho(t, x)) > 0$, for all $t \in [0, t_{\max}]$, of the initial-boundary value problem (5), (6), (7), (8), (9), (15), (16), (17), (18). The solution $\rho : [0, t_{\max}] \times [0, 1] \rightarrow (0, +\infty)$ satisfies for all $i \geq 0$ with $t_i < t_{\max}$ and $t \in [t_i, \min(t_{i+1}, t_{\max}))$:

$$\rho(t, x) = \begin{cases} \rho \left(t_i, x - \int_{t_i}^t v(s) ds \right) & \text{if } 1 \geq x > \int_{t_i}^t v(s) ds \\ \frac{u_i}{v(\tilde{t}(t, x))} & \text{if } 0 \leq x \leq \int_{t_i}^t v(s) ds \end{cases}, \tag{23}$$

where $\tilde{t}(t, x) \in [t_i, t]$ is the unique solution of the equation

$$x = \int_{\tilde{t}(t, x)}^t v(s) ds, \tag{24}$$

$$v(t) = \lambda(W(t)). \tag{25}$$

The uniqueness of solution can be established considering the function $\kappa(r) = \int_r^t v(s)ds$, defined for $r \in [0, t]$. Since $(t, x) \in \Phi$ with

$$\Phi := \left\{ (t, x) \in [0, T] \times [0, 1], x \leq \int_0^t v(s)ds \right\},$$

it follows that $x \leq \kappa(0)$ and from the definition of $\kappa(r)$ one deduces that $\kappa(t) = 0 \leq x$. Notice that κ is C^1 on $[0, t]$ with $\frac{d\kappa(r)}{dr} = -v(r) < 0$ for all $r \in [0, t]$. Therefore, κ is strictly decreasing on $[0, t]$. By continuity of κ and knowing that $\kappa(t) \leq x \leq \kappa(0)$, it follows that there exists $r \in [0, t]$ with $\kappa(r) = x$. The fact that κ is strictly decreasing on $[0, t]$ implies the existence of a unique $r \in [0, t]$ with $\kappa(r) = x$.

Formula (23) is a consequence of (5)–(8). Notice that since $v(t) \leq \lambda(0)$ (a direct consequence of the fact that $\lambda \in C^1(\mathbb{R}_+; (0, +\infty))$ is a non-increasing function) and $t_{i+1} - t_i \leq \frac{1}{\lambda(0)}$ (a direct consequence of (17)), it follows that formula (23) is valid for all $t \in [t_i, \min(t_{i+1}, t_{\max}))$.

We next define for all $t \in [0, t_{\max})$:

$$V(t) = \sup_{0 \leq x \leq 1} \{\Psi(t, x) \exp(-\sigma x)\}. \tag{26}$$

Combining (23) and (26) we get for all $i \geq 0$ with $t_i < t_{\max}$ and $t \in [t_i, \min(t_{i+1}, t_{\max}))$:

$$\begin{aligned} & \max \left\{ \sup_{0 < \xi \leq 1 - \int_{t_i}^t v(s)ds} \left(\Psi(t_i, \xi) \right. \right. \\ & \quad \left. \left. \times \exp \left(-\sigma \left(\int_{t_i}^t v(s)ds + \xi \right) \right) \right), \right. \\ & \quad \left. \sup_{0 \leq x \leq \int_{t_i}^t v(s)ds} \left(\Psi(\tilde{t}(t, x), 0) \exp(-\sigma x) \right) \right\} \\ & \leq \max \left\{ \exp \left(-\sigma \int_{t_i}^t v(s)ds \right) V(t_i), \right. \\ & \quad \left. \sup_{t_i \leq \tau \leq t} \left(\Psi(\tau, 0) \exp \left(-\sigma \int_{\tau}^t v(s)ds \right) \right) \right\}. \tag{27} \end{aligned}$$

For the derivation of (27), we have used the fact that $\tilde{t}(t, x) \in [t_i, t]$ is continuously decreasing with respect to x with $\tilde{t}(t, 0) = t$ and $\tilde{t} \left(t, \int_{t_i}^t v(s)ds \right) = t_i$.

Combined with definition (26), the event-trigger (16), (17), (as well as continuity of $v(t) = \lambda(W(t))$, which implies continuity of $\rho(t, 0) = \frac{u_i}{v(t)}$ for $t \in [t_i, \min(t_{i+1}, t_{\max}))$) gives for all $i \geq 0$ with $t_i < t_{\max}$:

$$\Psi(\tau, 0) \leq \exp \left(-\sigma \int_{t_i}^{\tau} \lambda(W(s))ds \right) V(t_i), \tag{28}$$

for all $\tau \in [t_i, \min(t_{i+1}, t_{\max}))$.

Combining (27) and (28), we get for all $i \geq 0$ with $t \in [t_i, \min(t_{i+1}, t_{\max}))$ and $t_i < t_{\max}$:

$$V(t) \leq \exp \left(-\sigma \int_{t_i}^t v(s)ds \right) V(t_i). \tag{29}$$

Definition (26) and inequality (29) show that $\|\rho[t]\|_{\infty}$ is bounded on $[t_i, \min(t_{i+1}, t_{\max}))$. Consequently, Theorem 1 guarantees that $t_{i+1} < t_{\max}$. Therefore, $t_{\max} = \lim_{i \rightarrow +\infty} (t_i)$.

Notice that the notion of solution that we have adopted guarantees that (29) is also valid for $t = t_{i+1}$. Applying (29) inductively, we get for all $t \in [0, \lim_{i \rightarrow +\infty} (t_i))$:

$$V(t) \leq \exp \left(-\sigma \int_0^t v(s)ds \right) V(0). \tag{30}$$

Since $v(t) = \lambda(W(t)) \geq 0$ for all $t \in [0, \lim_{i \rightarrow +\infty} (t_i))$, from (30) the following inequality holds

$$V(t) \leq V(0), \quad \forall t \in \left[0, \lim_{i \rightarrow +\infty} (t_i) \right). \tag{31}$$

Eqs. (7), (8), (15) and (18) with $i = 0$, imply that $\rho(0, 0) = \rho_s$ or equivalently that

$$\Psi(0, 0) = 0. \tag{32}$$

Applying (26), for $t = 0$ and using (9), and (32), we obtain:

$$\begin{aligned} V(0) &= \sup_{0 < x \leq 1} \{\Psi_0(x) \exp(-\sigma x)\} \\ &\leq \sup_{0 < x \leq 1} \Psi_0(x). \end{aligned} \tag{33}$$

On the other hand, definition (26) implies for all $t \in [0, \lim_{i \rightarrow +\infty} (t_i))$

$$\begin{aligned} V(t) &= \sup_{0 \leq x \leq 1} (\Psi(t, x) \exp(-\sigma x)) \\ &\geq \exp(-\sigma) \sup_{0 \leq x \leq 1} \Psi(t, x). \end{aligned} \tag{34}$$

Combining (31), (33) and (34), we get the following estimate for all $t \in [0, \lim_{i \rightarrow +\infty} (t_i))$:

$$\sup_{0 \leq x \leq 1} \Psi(t, x) \leq \exp(\sigma) \sup_{0 < x \leq 1} \Psi_0(x). \tag{35}$$

Definition (22) in conjunction with (6), (35) and the facts that $v(t) = \lambda(W(t))$, $\lambda \in C^1(\mathbb{R}_+; (0, +\infty))$ is a non-increasing function imply that $v(t) \geq c(\rho_0)$ for all $t \in [0, \lim_{i \rightarrow +\infty} (t_i))$. Estimate (20) for $t \in [0, \lim_{i \rightarrow +\infty} (t_i))$ is a direct consequence of estimate (30), definition (26) and the fact that $v(t) \geq c(\rho_0)$ for all $t \in [0, \lim_{i \rightarrow +\infty} (t_i))$.

The rest of the proof is devoted to the proof of (21) which also shows that $\lim_{i \rightarrow +\infty} (t_i) = +\infty$. Define for all $t \in [t_i, t_{i+1})$ and $i \geq 0$:

$$z(t) = \exp \left(\sigma \int_{t_i}^t \lambda(W(s)) ds \right) \ln \left(\frac{\rho(t, 0)}{\rho_s} \right) \tag{36}$$

By virtue of (36), (7), (8) and (18), we get for all $t \in [t_i, t_{i+1})$ and $i \geq 0$:

$$z(t) = \exp \left(\sigma \int_{t_i}^t \lambda(W(s)) ds \right) \ln \left(\frac{\lambda(W(t_i))}{\lambda(W(t))} \right) \tag{37}$$

Eq. (37) implies for all $i \geq 0$ and for $t \in [t_i, t_{i+1})$ almost everywhere:

$$\begin{aligned} \dot{z}(t) &= -\lambda'(W(t)) (\rho(t, 0) - \rho(t, 1)) \\ &\quad \times \exp \left(\sigma \int_{t_i}^t \lambda(W(s)) ds \right) + \sigma \lambda(W(t)) z(t). \end{aligned} \tag{38}$$

Differential equation (38) is a direct consequence of the fact that for $t \in [t_i, t_{i+1})$

$$W(t) = (t - t_i)u_i + \int_0^{a(t)} \rho(t, x)dx,$$

with

$$a(t) = 1 - \int_{t_i}^t \lambda(W(s))ds. \tag{39}$$

The previous equations and the fact that $\rho[t_i]$ is piecewise continuous show that the mapping $[t_i, t_{i+1}) \ni t \rightarrow W(t)$ is continuous and piecewise continuously differentiable for $t \in [t_i, t_{i+1})$. As a consequence of (37), we also obtain that the mapping $[t_i, t_{i+1}) \ni t \rightarrow z(t)$ is continuous and piecewise continuously differentiable for $t \in [t_i, t_{i+1})$.

It follows from the triangle inequality, (36), (38), the fact that $\lambda \in C^1(\mathbb{R}_+; (0, +\infty))$ is a non-increasing function and the fact that $|\lambda'(s)| \leq K$ for all $s \geq 0$, that the following inequality holds for all $i \geq 0$ and $t \in [t_i, t_{i+1})$ almost everywhere:

$$|\dot{z}(t)| \leq K |\rho(t, 0) - \rho(t, 1)| \exp\left(\sigma \int_{t_i}^t \lambda(W(s)) ds\right) + \sigma \lambda(0) |z(t)| \tag{40}$$

$$\leq K |\rho(t, 0) - \rho_s| \exp\left(\sigma \int_{t_i}^t \lambda(W(s)) ds\right) + K |\rho(t, 1) - \rho_s| \exp(\sigma \lambda(0)(t - t_i)) + \sigma \lambda(0) |z(t)|. \tag{41}$$

Formulas (23), (26) and the fact that $|\exp(x) - 1| \leq \exp(|x|) - 1$ for all $x \in \mathbb{R}$, leads to the following estimates

$$|\rho(t, 1) - \rho_s| \leq \|\rho[t_i] - \rho_s\|_\infty, \\ \|\rho[t_i] - \rho_s\|_\infty \leq \rho_s (\exp(\exp(\sigma)V(t_i)) - 1).$$

Moreover, the fact that $|\exp(x) - 1| \leq |x| \exp(|x|)$ for all $x \in \mathbb{R}$, shows that

$$|\rho(t, 0) - \rho_s| \leq \rho_s \exp(\Psi(t, 0)) \Psi(t, 0).$$

Using the previous inequalities in conjunction with (26), (29), (41) and (36), we get the following inequality for all $i \geq 0$ and $t \in [t_i, t_{i+1})$ almost everywhere:

$$|\dot{z}(t)| \leq (K \rho_s \exp(V(t_i)) + \sigma \lambda(0)) |z(t)| + K \rho_s (\exp(\exp(\sigma)V(t_i)) - 1) \exp(\sigma \lambda(0)(t - t_i)). \tag{42}$$

Using (42), the facts that $z(t_i) = 0$, $t_{i+1} - t_i \leq \frac{1}{\lambda(0)}$ (a direct consequence of (17)), and the Gronwall–Bellman lemma, we get for all $i \geq 0$ and $t \in [t_i, t_{i+1})$:

$$|z(t)| \leq (\exp(\exp(\sigma)V(t_i)) - 1) \times \exp(\sigma) \frac{\exp(K \rho_s \exp(V(t_i))(t - t_i)) - 1}{\exp(V(t_i))}. \tag{43}$$

It follows from (43) that the inequality $|z(t)| \leq V(t_i)$ holds for all $t \in [t_i, t_{i+1})$, provided that

$$t_{i+1} - t_i \leq \tilde{T}(V(t_i)), \tag{44}$$

where

$$\tilde{T}(s) := \begin{cases} \frac{1}{K \rho_s} \exp(-s) \ln\left(1 + \frac{s \exp(s-\sigma)}{\exp(\exp(\sigma)s)-1}\right) & \text{if } s > 0 \\ \frac{1}{K \rho_s} \ln(1 + \exp(-2\sigma)) & \text{if } s = 0 \end{cases} \tag{45}$$

with $\tilde{T}(0) = \lim_{s \rightarrow 0^+} \tilde{T}(s)$.

Using (16), (17), (36) and (45), we conclude that

$$t_{i+1} - t_i \geq \min\left(\frac{1}{\lambda(0)}, \tilde{T}(V(t_i))\right). \tag{46}$$

Continuity and positivity of the function $\tilde{T}(s)$ defined by (45), implies that there exists a non-increasing function $T : \mathbb{R}_+ \rightarrow (0, +\infty)$ such that

$$\min\left(\frac{1}{\lambda(0)}, \tilde{T}(s)\right) \geq T(s),$$

for all $s \geq 0$. Inequality (21) is a consequence of the previous inequality, (46), (30) and definition (26). The proof is complete. \triangleleft

Remark 3. Estimate (21) guarantees that no Zeno behavior can appear for the closed-loop system (5), (6), (7), (8), (15), (16), (17), (18) even if a positive minimum dwell-time is not explicitly established. The proof of Theorem 2 provides an estimate for the function $T : \mathbb{R}_+ \rightarrow (0, +\infty)$.

4.2. Robustness with respect to the event sequence

Let $\rho_0 \in PC^1([0, 1])$ with $\inf_{x \in (0, 1)} (\rho_0(x)) > 0$ be given and define:

$$G(\rho_0) := \min\left(\frac{1}{\lambda(0)}, r\right), \tag{47}$$

where

$$r := \inf\left\{ \tau > 0 : \Psi(\tau, 0) > \exp\left(-\sigma \int_0^\tau \lambda(W(s)) ds\right) \times \sup_{0 < x \leq 1} (\Psi_0(x) \exp(-\sigma x)) \right\}, \tag{48}$$

where $\rho[t] \in PC^1([0, 1])$ and $\inf_{x \in [0, 1]} (\rho(t, x)) > 0$ for all $t \geq 0$, is the solution of (5), (6), (7), (9) with $u(t) \equiv \rho_s \lambda\left(\int_0^1 \rho_0(x) dx\right)$. Definitions (47), (48) imply that the event-triggered control (15), (16), (17), (18) satisfies the following relation for all $i \geq 0$:

$$t_{i+1} = t_i + G(\rho[t_i]) \tag{49}$$

Theorem 2 (and particularly inequality (11)) shows that for every initial condition $\rho_0 \in PC^1([0, 1])$ with $\inf_{x \in (0, 1)} (\rho_0(x)) > 0$, the sequence of events $\{t_i : i = 0, 1, 2, \dots\}$ with $t_0 = 0$ is a diverging sequence, i.e., $\lim_{i \rightarrow +\infty} (t_i) = +\infty$. However, there is an infinite number of diverging increasing sequences $\{t_i : i = 0, 1, 2, \dots\}$ with $t_0 = 0$ for which $t_{i+1} - t_i \leq G(\rho[t_i])$ for all $i \geq 0$. For these sequences, the controller acts (through (9)) before an event occurs. The following result extends the result of Theorem 2 and guarantees robustness with respect to the event sequence.

Theorem 3. Suppose that there exists a constant $K > 0$ such that $|\lambda'(s)| \leq K$ for all $s \geq 0$. Let $\sigma > 0$ be a given parameter. Then for every $\rho_0 \in PC^1([0, 1])$ with $\inf_{x \in (0, 1)} (\rho_0(x)) > 0$ and for every increasing sequence of times $\{t_i : i = 0, 1, 2, \dots\}$ with $t_0 = 0$, $\lim_{i \rightarrow +\infty} (t_i) = +\infty$ that satisfies

$$t_{i+1} - t_i \leq G(\rho[t_i]), \text{ for all } i \geq 0, \tag{50}$$

the solution $\rho : \mathbb{R}_+ \times [0, 1] \rightarrow (0, +\infty)$ with $\rho[t] \in PC^1([0, 1])$ and $\inf_{x \in [0, 1]} (\rho(t, x)) > 0$ for all $t \geq 0$ of the initial-boundary value problem (5), (6), (7), (8), (9), (18) satisfies estimate (20).

Proof of Theorem 3. The proof is essentially the same as the first part of the proof of Theorem 2. The only difference is to notice that the event-trigger (16), (17), (as well as continuity of $v(t) = \lambda(W(t))$, which implies continuity of $\rho(t, 0) = \frac{u_i}{v(t)}$ for $t \in [t_i, \min(t_{i+1}, t_{\max}))$) gives for all $i \geq 0$ with $t_i < t_{\max}$ when combined with definitions (26), (47), (48):

$$\Psi(\tau, 0) \leq \exp\left(-\sigma \int_{t_i}^\tau \lambda(W(s)) ds\right) V(t_i), \tag{51}$$

for all $\tau \in [t_i, \min(t_{\max}, t_i + G(\rho[t_i]))]$. Inequality (51) replaces inequality (28). Using (51) we end up with inequality (35) exactly as in the proof of Theorem 2. The rest of the proof of Theorem 2 is not needed because it is not needed to show inequality (21) and it is assumed that $\lim_{i \rightarrow +\infty} (t_i) = +\infty$. The proof is complete. \triangleleft

4.3. Sampled-data stabilization with robustness with respect to the sampling schedule

Theorem 3 is important because it shows that the controller (18) can be implemented in various ways. For example, we can

implement the controller (18) in a sample-and-hold fashion for an appropriate sampling period. This is shown by the following result.

Theorem 4. Suppose that there exists a constant $K > 0$ such that $|\lambda'(s)| \leq K$ for all $s \geq 0$. Let $\sigma > 0$ be a given parameter. Then, for every $R > 0$ there exists $\tau > 0$ such that for every $\rho_0 \in PC^1([0, 1])$ with $\inf_{x \in (0, 1)}(\rho_0(x)) > 0$, $\sup_{0 < x \leq 1} \Psi_0(x) \leq R$ and for every increasing sequence of times $\{t_i : i = 0, 1, 2, \dots\}$ with $t_0 = 0$, $\lim_{i \rightarrow +\infty} (t_i) = +\infty$ that satisfies

$$t_{i+1} - t_i \leq \tau, \text{ for all } i \geq 0, \tag{52}$$

the solution $\rho : \mathbb{R}_+ \times [0, 1] \rightarrow (0, +\infty)$ with $\rho[t] \in PC^1([0, 1])$ and $\inf_{x \in [0, 1]}(\rho(t, x)) > 0$ for all $t \geq 0$ of the initial-boundary value problem (5), (6), (7), (8), (9), (18) satisfies estimate (20).

The sample-and-hold implementation of the controller (18) does not require continuous measurement of the state. On the other hand, the time $\tau > 0$ is (in general) much smaller than $G(\rho[t_i])$, which implies that the control action must be updated much more frequently in the sampled-data case than in the event-triggered case.

Proof of Theorem 4. The proof of Theorem 2 and definitions (47), (48) actually show that there exists a non-increasing function $T : \mathbb{R}_+ \rightarrow (0, +\infty)$ such that for every $R > 0$ and for every $\rho \in PC^1([0, 1])$ with $\inf_{x \in (0, 1)}(\rho(x)) > 0$ and $\sup_{0 < x \leq 1} \Psi(x) \leq R$, the following inequality holds:

$$G(\rho) \geq T(R). \tag{53}$$

Setting $\tau := T(R)$ and repeating the proof of Theorem 2 with (51) replacing (28), we are in a position to show that inequality (35) holds for every increasing sequence of times $\{t_i : i = 0, 1, 2, \dots\}$ with $t_0 = 0$, $\lim_{i \rightarrow +\infty} (t_i) = +\infty$ that satisfies (52). The proof is complete. \triangleleft

5. Simulation results

5.1. Event-triggered boundary control law

We simulate closed-loop system consisting of (5), (6), (7), (9) together with the even triggered controller (15)–(18). The nonlocal propagation speed of PDE (5) is defined as

$$\lambda(W) = \frac{1}{1+W},$$

where W is given by (6). The initial condition is set to $\rho_0(x) = 6 + \sin(\pi x)$ and the equilibrium density is defined as $\rho_s = 1$. The event generator is computed for two values of σ , namely, $\sigma = 0.02$ and $\sigma = 0.006$. Here, the number of updating times of the control signal is an increasing function of the parameter σ . Fig. 2 shows the evolution of the influx of parts that is the boundary control action while Fig. 3 reflects the dynamics of the density at the input with the event-triggered instants. From Figs. 3 and 4, it can be viewed that both the input and the output density are stabilized at the desired uniform equilibrium $\rho_s = 1$ for both values of σ . However, the greater is the value of σ , the faster is the convergence rate due to the increasing number of execution of the control task (18). Moreover, Fig. 5, which represents the L^2 -norm of $\Psi(t, x)$, shows that $\rho(t, x)$ converges to the equilibrium in a L^2 sense. This statement is confirmed by Figs. 6 and 7 which shows the evolution in time of the distributed density $\rho(t, x)$ for both values of σ .

For a set of initial conditions defined as $\rho_0(x) = 6 + \sin(\pi x) + l \times x^4$, $l = 1, \dots, 100$, Fig. 8 represents the statistics on the inter-execution times under the event-triggered policy (16), (17) for

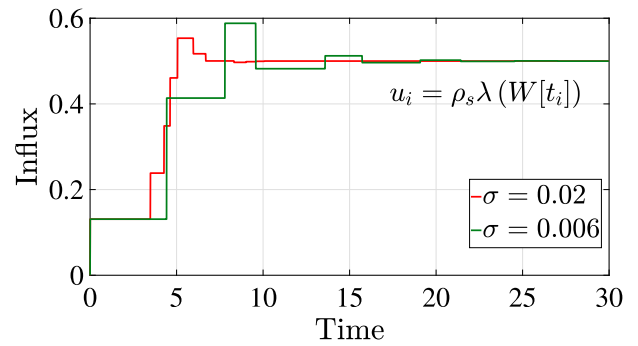


Fig. 2. Influx dynamics: event-triggered control action at the boundary $x = 0$.

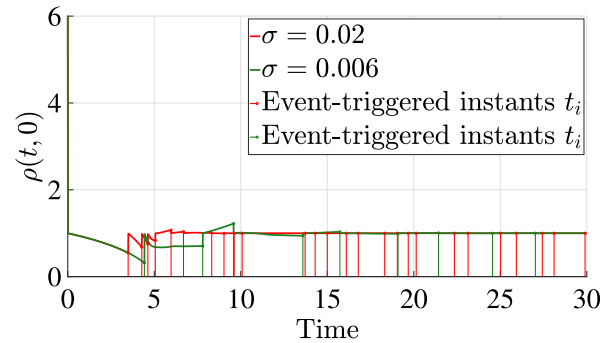


Fig. 3. Density evolution in time at the controlled boundary and event-trigger instants t_i .

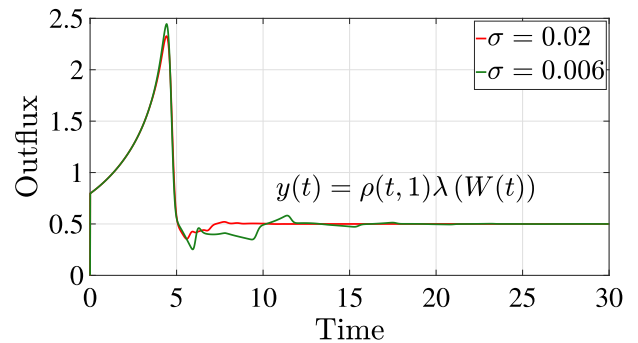


Fig. 4. Outflux dynamics at the uncontrolled boundary $x = 1$.

$\sigma = 0.02$ and $\sigma = 0.006$, in a logarithmic scale. For a fast sampling, $\sigma = 0.02$, the inter-execution time $\tau = t_{i+1} - t_i$ is predominantly of order 0.5 while for slow sampling τ is of order 2. These values can be used as indicative (but possibly conservative) choices of the sampling periods applying a sampled data control approach knowing the robustness of the event-based control with respect to the triggering policy.

5.2. Sampled-data boundary control law

To illustrate the robustness of the control action concerning the sampling schedule, we apply the controller (18) with a periodically updated control action. Here, the simulation is performed under the previous initial condition with an identical function λ . Two sampling periods are considered, namely, $T = 1$ and $T = 3$, as shown in Fig. 9 motivated by the statistics in Fig. 8. The results

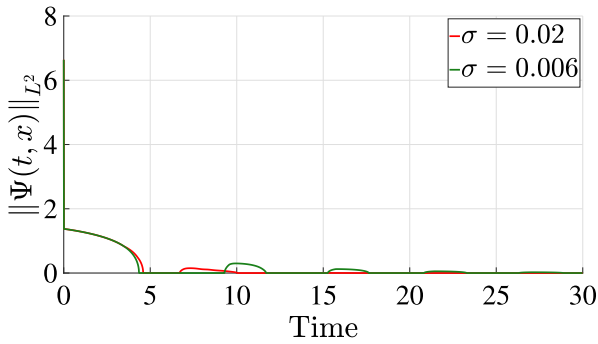


Fig. 5. L^2 norm of the distributed density deviation for different values of σ .

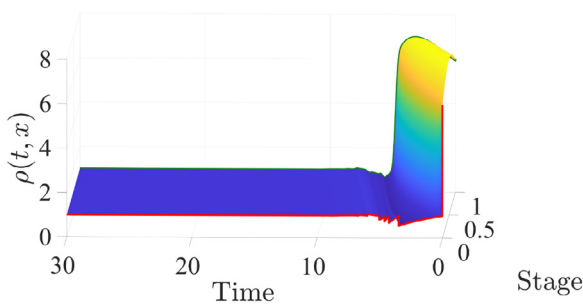


Fig. 6. The dynamics of the distributed density function for $\sigma = 0.02$.

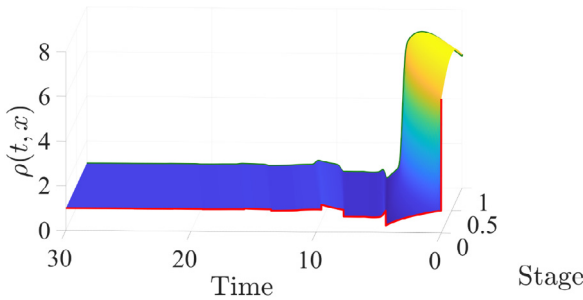


Fig. 7. The dynamics of the distributed density function for $\sigma = 0.006$.

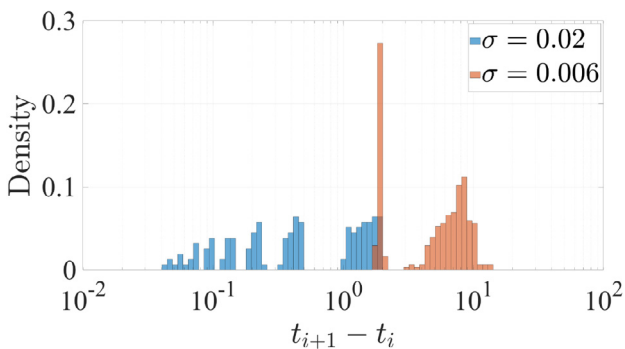


Fig. 8. The inter-execution time $\sigma = 0.02$ and $\sigma = 0.006$: statistics with 200 initial conditions.

obtained in Figs. 10 and 11 prove that the input density converge to the uniform setpoint $\rho_s = 1$ and the output flux is also stabilized at the equilibrium. As expected, the L^2 norm of $\Psi(t, x)$ tends to zero (Fig. 12), and the distributed density function are

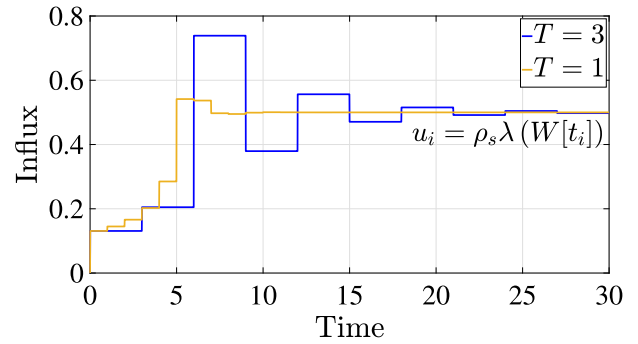


Fig. 9. Influx dynamics with sampled data boundary control action.

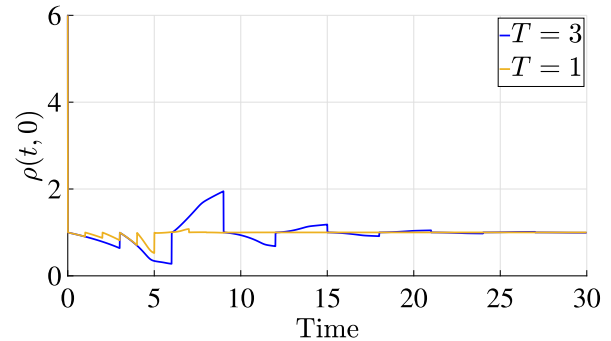


Fig. 10. The distributed density evolution in time at the controlled boundary ($x = 0$) with sampled data control.

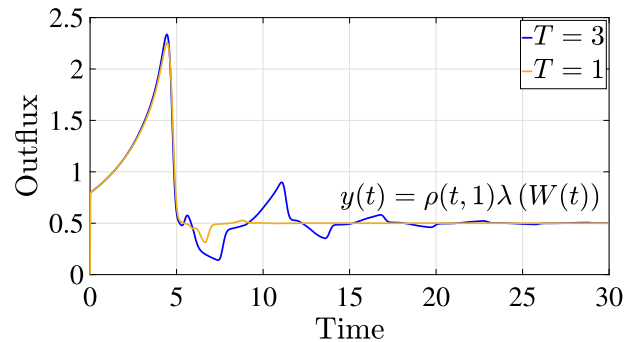


Fig. 11. Outflux dynamics at the uncontrolled boundary $x = 1$ with sampled data control.

stabilized to ρ_s for both the considered sampling periods (Figs. 13 and 14). As for the event-triggered control, one can notice that fast sampling ($T = 1$) enabled better closed-loop performance.

Moreover, considering a sampling period $T = 3$, both the ETC ($\sigma = 0.02$ and $\sigma = 0.006$) simulations converge faster than the sampled-data controller, which shows that the ETC control scheme outperforms the periodic sampling control approach.

6. Concluding remarks

In this paper, an even-triggered control algorithm is developed to stabilize the continuum model of a highly re-entrant manufacturing system. The robustness of the proposed controller with respect to the sampling policy is proven to enable the implementation of the classical sampled-data controller with a cyclic update of the control action. Our controller requires the measurement of the distributed density and developing an output feedback event-triggered controller that only relies on boundary or point measurements will be considered in our future works.

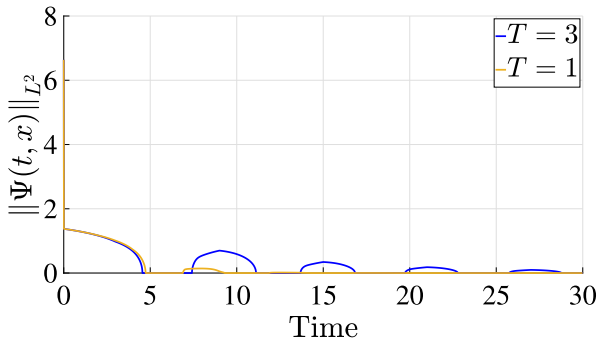


Fig. 12. L^2 norm of the distributed density deviation for the two sampling periods.

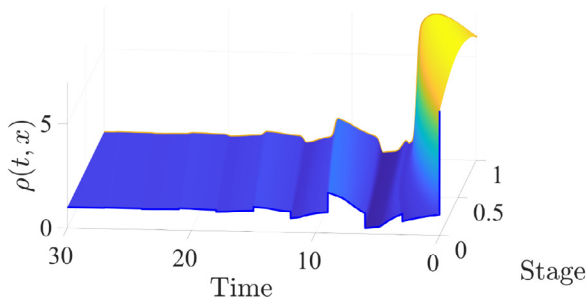


Fig. 13. The distributed density dynamics with sampling period $T = 3$.

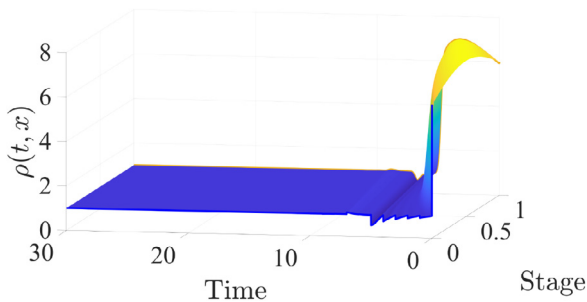


Fig. 14. The distributed density dynamics with sampling period $T = 1$.

Appendix

Proof of Theorem 1. Let $\rho_0 \in PC^1([0, 1])$, $u > 0$ and $\inf_{x \in (0, 1)} (\rho_0(x)) > 0$ be given (arbitrary). Define

$$\rho_{\max} := \sup_{x \in (0, 1)} (\rho_0(x)), \quad \rho_{\min} := \inf_{x \in (0, 1)} (\rho_0(x)). \tag{54}$$

Consider for each $T \in (0, \frac{1}{\lambda(0)}]$ the mapping $P : S \rightarrow S$, where

$$S = \left\{ W \in C^0([0, T]) : 0 \leq W(t) \leq \rho_{\max} + \frac{u}{\lambda(0)}, t \in [0, T] \right\}, \tag{55}$$

which maps every $W \in S$ to the function $P(W) \in S$ given by the formula

$$P(W)(t) = tu + \int_0^{a(t)} \rho_0(x) dx, \text{ for } t \in [0, T], \tag{56}$$

where

$$a(t) = 1 - \int_0^t \lambda(W(s)) ds, \text{ for } t \in [0, T]. \tag{57}$$

For all $W, V \in S$ for the mapping $P : S \rightarrow S$ defined by (56), (57), we have that

$$|P(W)(t) - P(V)(t)| = \left| \int_{\bar{a}(t)}^{a(t)} \rho_0(x) dx \right|, \tag{58}$$

where

$$\bar{a}(t) = 1 - \int_0^t \lambda(V(s)) ds, \text{ for } t \in [0, T]. \tag{59}$$

From (54) and (57)–(59), the following estimate holds

$$|P(W)(t) - P(V)(t)| \leq \rho_{\max} \left| \int_0^t [\lambda(W(s)) - \lambda(V(s))] ds \right|, \tag{60}$$

which leads to

$$|P(W)(t) - P(V)(t)| \leq t \rho_{\max} \sup_{s \geq 0} \lambda'(s) \left(\max_{0 \leq t \leq T} |W(t) - V(t)| \right). \tag{61}$$

Hence, the following inequality holds

$$\max_{t \in [0, T]} (|P(W)(t) - P(V)(t)|) \leq \rho_{\max} K T \left(\max_{t \in [0, T]} |W(t) - V(t)| \right), \tag{62}$$

where $K > 0$ is the constant for which $|\lambda'(s)| \leq K$ for all $s \geq 0$.

Therefore, for

$$T = \frac{1}{\lambda(0) + K \rho_{\max}},$$

the mapping $P : S \rightarrow S$ is a contraction and Banach's fixed-point theorem implies the existence of a unique $W \in S$ such that

$$W(t) = tu + \int_0^{a(t)} \rho_0(x) dx, \text{ for } t \in [0, T], \tag{63}$$

where $a(t)$ is given by (57). Notice that definitions (54) and Eq. (63) as well as the fact that $\lambda(s) \leq \lambda(0)$ for all $s \geq 0$, imply the following estimate:

$$W(t) \geq tu + \rho_{\min}(1 - \lambda(0)t) > 0, \text{ for } t \in [0, T] \tag{64}$$

Next define the functions $v : [0, T] \rightarrow (0, +\infty)$, $\rho : [0, T] \times [0, 1] \rightarrow (0, +\infty)$ by means of the equations

$$v(t) = \lambda(W(t)), \text{ for } t \in [0, T], \tag{65}$$

$$\rho(t, x) = \begin{cases} \rho_0 \left(x - \int_0^t v(s) ds \right) & \text{if } \int_0^t v(s) ds < x \leq 1 \\ \frac{u}{v(\tilde{t}(t, x))} & \text{if } 0 \leq x \leq \int_0^t v(s) ds \end{cases}, \tag{66}$$

where $\tilde{t}(t, x) \in [0, t]$ is the unique solution of (24) for all

$$(t, x) \in \Phi := \left\{ (t, x) \in [0, T] \times [0, 1], x \leq \int_0^t v(s) ds \right\}.$$

Notice that $\rho[t] \in PC^1([0, 1])$ for each $t \in [0, T]$ and that

$$\begin{aligned} \int_0^1 \rho(t, x) dx &= \int_0^{1-a(t)} \rho(t, x) dx + \int_{1-a(t)}^1 \rho(t, x) dx, \\ &= u \int_0^{1-a(t)} \frac{dx}{v(\tilde{t}(t, x))} \\ &\quad + \int_{1-a(t)}^1 \rho_0(x + a(t) - 1) dx, \\ &= ut + \int_0^{a(t)} \rho_0(x) dx = W(t). \end{aligned} \tag{67}$$

For the derivation of (67), we have used formulas (57), (65), (66), (63) and the fact that following (24)

$$\frac{\partial \tilde{t}}{\partial x}(t, x) = -\frac{1}{v(\tilde{t}(t, x))}, \tag{68}$$

for all $(t, x) \in \Phi$, where

$$\Phi := \left\{ (t, x) \in [0, T] \times [0, 1], x \leq \int_0^t v(s) ds \right\},$$

with

$$\tilde{t}(t, 0) = t \quad \text{and} \quad \tilde{t}\left(t, \int_0^t v(s) ds\right) = 0. \tag{69}$$

We next repeat the construction with ρ_0 replaced by $\rho[T]$. We can construct functions $\tilde{v} : [0, T'] \rightarrow (0, +\infty)$, $\tilde{\rho} : [0, T'] \times [0, 1] \rightarrow (0, +\infty)$, $\tilde{W} \in C^0([0, T'])$ with

$$0 \leq \tilde{W}(t) \leq \|\rho[T]\|_\infty + \frac{u}{\lambda(0)} \text{ for } t \in [0, T'],$$

and

$$T' = \frac{1}{\lambda(0) + K \|\rho[T]\|_\infty}$$

such that

$$\begin{aligned} \tilde{W}(t) &= \int_0^1 \tilde{\rho}(t, x) dx, \\ \tilde{v}(t) &= \lambda(\tilde{W}(t)), \\ \tilde{W}(t) &= tu + \int_0^{a(t)} \rho(T, x) dx, \\ \tilde{a}(t) &= 1 - \int_0^t \lambda(\tilde{W}(s)) ds. \end{aligned}$$

It is a matter of straightforward calculations to verify that the extensions of ρ, v, W given by the formulas for $t \in (T, T + T']$, $x \in [0, 1]$:

$$\begin{aligned} \rho(t, x) &= \tilde{\rho}(t - T, x), \\ W(t) &= \tilde{W}(t - T), \\ v(t) &= \tilde{v}(t - T) \end{aligned}$$

satisfy the following equations for all $t \in [0, T + T']$, $x \in [0, 1]$:

$$v(t) = \lambda(W(t)), \tag{70}$$

$$\rho(t, x) = \begin{cases} \rho_0\left(x - \int_0^t v(s) ds\right) & \text{if } \int_0^t v(s) ds < x \leq 1 \\ \frac{u}{v(\tilde{t}(t, x))} & \text{if } 0 \leq x \leq \int_0^t v(s) ds \end{cases}, \tag{71}$$

$$W(t) = \int_0^1 \rho(t, x) dx, \tag{72}$$

where $\tilde{t}(t, x) \in [0, t]$ is the unique solution of (24) for all $(t, x) \in \Phi$ with

$$\Phi := \left\{ (t, x) \in [0, T + T'] \times [0, 1], x \leq \int_0^t v(s) ds \right\}.$$

The construction can be repeated ad infinitum and thus we obtain functions $v : [0, t_{\max}) \rightarrow (0, +\infty)$, $\rho : [0, t_{\max}) \times [0, 1] \rightarrow (0, +\infty)$, $W \in C^0([0, t_{\max}))$ that satisfy (70), (71), (72) for all $t \in [0, t_{\max})$, $x \in [0, 1]$. Moreover, if $t_{\max} < +\infty$ then $\limsup_{t \rightarrow t_{\max}^-} (\|\rho[t]\|_\infty) = +\infty$. Furthermore, $\rho[t] \in PC^1([0, 1])$ for each $t \in [0, t_{\max})$.

Finally, formulas (70), (71) and the facts that $\lambda(s) \leq \lambda(0)$ for all $s \geq 0$, $\inf_{x \in (0, 1]} (\rho_0(x)) > 0$ imply that $\inf_{x \in [0, 1]} (\rho(t, x)) > 0$ for each $t \in [0, t_{\max})$. Notice that Eqs. (6), (7), (9) hold for $t \in [0, t_{\max})$ with $u(t) \equiv u > 0$. Moreover, the following holds:

$$\begin{aligned} W(t) &= (t - \tilde{t}(t, \min\{1, 1 - a(t)\})) u \\ &\quad + \int_0^{\max\{0, a(t)\}} \rho_0(\xi) d\xi, \text{ for } t \in [0, t_{\max}). \end{aligned} \tag{73}$$

The proof of (73) is established considering the following two cases:

- **Case 1:** $\int_0^t v(s) ds < 1$ or $a(t) \in (0, 1]$.
By virtue of (70), (71), (72) and (57) we get the following equality for $t \in [0, t_{\max})$

$$\begin{aligned} W(t) &= \int_0^1 \rho(t, x) dx \\ &= \int_0^{1-a(t)} \rho(t, x) dx + \int_{1-a(t)}^1 \rho_0(x) dx \end{aligned} \tag{74}$$

Hence,

$$W(t) = u \int_0^{1-a(t)} \frac{1}{v(\tilde{t}(t, x))} dx + \int_{1-a(t)}^1 \rho_0(x + a(t) - 1) dx. \tag{75}$$

Since (68) holds for all $(t, x) \in \Phi$, by using the change of variable $\xi = \tilde{t}(t, x)$ we obtain the following equation

$$\int_0^{1-a(t)} \frac{1}{v(\tilde{t}(t, x))} dx = \int_{\tilde{t}(t, 0)}^{\tilde{t}(t, 1-a(t))} d\xi. \tag{76}$$

Using (57), (75), (69), (70) and (76), we have for $t \in [0, t_{\max})$:

$$W(t) = ut + \int_{1-a(t)}^1 \rho_0(x + a(t) - 1) dx \tag{77}$$

$$= ut + \int_0^{a(t)} \rho_0(s) ds. \tag{78}$$

Knowing that $\min\{1, 1 - a(t)\} = 1 - a(t)$ (a consequence of the fact that $a(t) \in (0, 1]$ and since $\tilde{t}(t, 1 - a(t)) = 0$ (a consequence of (57), (69) and (70)), Eq. (78) shows that (73) holds.

- **Case 2:** $\int_0^t v(s) ds \geq 1$ or $a(t) \leq 0$.
By virtue of (70), (71), (72) and (57) we get for $t \in [0, t_{\max})$

$$W(t) = \int_0^1 \rho(t, x) dx = u \int_0^1 \frac{1}{v(\tilde{t}(t, x))} dx. \tag{79}$$

Since (68) holds for all $(t, x) \in \Phi$ we obtain by using the change of variable $\xi = \tilde{t}(t, x)$ the following equation

$$\int_0^1 \frac{1}{v(\tilde{t}(t, x))} dx = \int_{\tilde{t}(t, 1)}^{\tilde{t}(t, 0)} d\xi. \tag{80}$$

Using (57), (79), (69), (70) and (80), we have for $t \in [0, t_{\max})$:

$$W(t) = u(t - \tilde{t}(t, 1)). \tag{81}$$

Since $\min\{1, 1 - a(t)\} = 1$ and $\max\{0, a(t)\} = 0$ (both consequences of the fact that $a(t) \leq 0$), Eq. (81) shows that (73) holds.

Eq. (71) implies that ρ is not C^1 at specific points:

- The points $(t, x) \in [0, t_{\max}] \times [0, 1]$ for which

$$\xi_i = x - \int_0^t v(s)ds, \tag{82}$$

where $\xi_i \in [0, 1]$ ($i = 0, \dots, N$) are the points (in increasing order with $\xi_0 = 0$) for which $\rho_0 \in C^1([0, 1] \setminus \{\xi_0, \dots, \xi_N\})$ due to the lack of regularity of the initial condition $\rho_0(x)$.

- On the other hand, Eq. (71), shows that ρ may not be C^1 at the points $(t, x) \in [0, t_{\max}] \times [0, 1]$ for which $\tilde{t}(t, x)$ is equal to a time where W is not C^1 . Knowing that (73) can be rewritten as

$$W(t) = \begin{cases} ut + \int_0^{a(t)} \rho_0(x) dx & \text{for } a(t) \geq 0 \\ (t - \tilde{t}(t, 1))u & \text{for } a(t) < 0 \end{cases}, \tag{83}$$

we deduce that the W is not C^1 at the times where $a(t) = 0$ and $a(t)$ is equal to a point where ρ_0 is discontinuous. Clearly, all discontinuity points of ρ_0 are included in the set $\{\xi_0, \dots, \xi_N\}$ and therefore the times $\tau \in [0, t_{\max}]$ with $a(\tau) \in \{\xi_0, \dots, \xi_N\}$ are the times of concern. Consequently, the points $(t, x) \in [0, t_{\max}] \times [0, 1]$ for which $\tilde{t}(t, x)$ is equal to a time where W is not C^1 are included in the set of all (t, x) for which $a(\tilde{t}(t, x)) \in \{\xi_0, \dots, \xi_N\}$.

Since $x = \int_{\tilde{t}(t,x)}^t v(s)ds$ and $a(t) = 1 - \int_0^1 v(s)ds$, it follows that

$$a(\tilde{t}(t, x)) = 1 + x - \int_0^1 v(s)ds \tag{84}$$

and the discontinuity occurs at the points $(t, x) \in [0, t_{\max}] \times [0, 1]$ satisfying

$$\xi_i = 1 + x - \int_0^1 v(s)ds, \quad i = 0, \dots, N. \tag{85}$$

Since

$$r_i(t) = \xi_i + \int_0^1 \lambda(W(s))ds = \xi_i + \int_0^1 v(s)ds,$$

for $i = 0, \dots, N$, it follows that $x = r_i(t) - 1$.

Finally, combining the sets defined by (11) and (12), we arrive at (10).

The fact that Eq. (5) holds for all $(t, x) \in [0, t_{\max}] \times [0, 1] \setminus \Omega$ is a direct consequence of formula (71), the above regularity properties for ρ, v, W and the fact that following (24)

$$\frac{\partial \tilde{t}}{\partial x}(t, x) = -\frac{1}{v(\tilde{t}(t, x))}, \quad \frac{\partial \tilde{t}}{\partial t}(t, x) = \frac{v(t)}{v(\tilde{t}(t, x))},$$

for all $(t, x) \in \Phi$,

$$\Phi := \left\{ (t, x) \in [0, t_{\max}] \times [0, 1], x \leq \int_0^t v(s)ds \right\}.$$

Uniqueness of solution is a consequence of Banach's fixed-point theorem: the fact that Eq. (73) has a unique solution $W \in C^0([0, t_{\max}])$. The solution of (73) is constructed step-by-step by using the mapping $P : S \rightarrow S$ defined by (56), (57) and Banach's fixed-point theorem guarantees that $P : S \rightarrow S$ has a unique fixed point.

The proof is complete. \square

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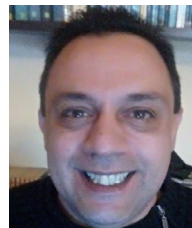
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